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SEGREGATION KINETICS OF PARTICLE RAPID GRAVITY FLOW

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Abstract: The problem of describing the segregation kinetics in a rapid gravity flow of particulate solids is analyzed. The experimental and analytical method of segregation coefficient determination during rapid gravity flow of particles is proposed. The method developed is used to analyze the kinetic behaviour of segregation in a rapid shear flow and to correct the equation of segregation kinetics suggested earlier [2]. The forecasting properties of the suggested model of segregation kinetics are investigated when this model is used to determine both the segregation dynamics of particulate mixtures and the velocities of the transverse displacement of single large or small particles in rapid gravity flows of particulate media.

List of symbols

c – test component concentration, kg·kg^{-1};
d – particle diameter, m;
d_b – mean diameter of bulk particle, m;
D_{dif} – quasi–diffusion coefficient, m^2·s^{-1};
D_m – migration coefficient, m^2·s^{-1};
F – frequency of collision calculated in term of one particle,·s^{-1};
K – coefficient of relative rate of segregation (constant of segregation kinetics), N^{-1}·s^{-1};
K_s – velocity of segregating component towards segregation flux (segregation coefficient), m·s^{-1};
k – coefficient of restitution at particle collisions;
j_s – segregation flux, kg·m^{-2}·s^{-1};
m – particle mass, kg;
M – momentum of force acting on the test particle, N·m;
ΔM – segregation driving force (nonuniformity parameter of particulate medium), N·m;
s – mean distance between particles, m;
u – mean velocity of particles in shear direction x, m·s^{-1};
$\bar{V}'$ – fluctuating component of the particle velocity, m·s^{-1};
(x, y) – Cartesian coordinats, Fig. 1.

Greek letters

ε – fraction of void volume (m^3·m^{-3});
ρ – particle density (kg·m^{-3});
ρ_b – bulk density (kg·m^{-3});
τ – time (s).

Indices

t – apply to test particle;
(x, y) – x, y components respectively.

1 Introduction

The rapid gravity flows of particulate solids are the most widespread type of rapid shear flows. Such kind of flow is accompanied by intensive segregation and mixing of nonuniform particles in consequence of their intensive interaction. At present the problem of forecasting such interaction effects has not been solved yet for many important cases.

Taking into account the existence of a number of segregation mechanisms [1] it is difficult now to suppose the creation of an universal physical model of segregation. Nevertheless at this situation it is very important to define general segregation mechanisms for the most common forms of particulate solids movement and to use them for the mathematical modeling of segregation dynamics. In the present paper the further development of segregation kinetics modeling based on the physical model of particles hydromechanical separation [2] is being carried out.

The equation of segregation dynamics in a steady two–dimensional shear flow, suggested in the work [3], may be formulated as follows

$$\frac{\partial c\rho_b}{\partial \tau} = -\frac{\partial uc\rho_b}{\partial x} + \frac{\partial}{\partial y}\left[\rho_b\left(D_{dif}\frac{\partial c}{\partial y} - D_m c\frac{\partial \ln s}{\partial y} - K\Delta Mc\left(1-c\right)\right)\right], \quad (1)$$

where the first item takes into account the convection flux of a test component in shear direction x; the second item is the flux of quasidiffusional mixing; the third item is the flux of quasidiffusional segregation or migration and the forth item takes into account the segregation flux.

According to eq. (1) the segregation occurs due to the following aspects of the nonuniformity of particulate medium: nonuniformity of particle properties; nonuniformity of the medium due to addition of one component to the other; nonuniformity of the spatial distribution of solid volume fraction.

Thus, the first two aspects characterize the local nonuniformity, while the third one characterizes the spatial nonuniformity of the medium.

In case of the rapid gravity flow of cohesionless nonelastic spherical particles the kinetic coefficients D_{dif} and D_m are calculated assuming a formal analogy between particulate media during rapid shear and a dense gas and taking into account the real physical features of particle interactions [2]. The coefficient of quasi-diffusional mixing is defined in the following way [2]

$$D_{\text{dif}} = 0,5s\overline{V}'. \quad (2)$$

The kinetic coefficient of migration, taking place due to the gradient of fraction of solid volume, is calculated for a binary mixture as follows [4]

$$D_m = \frac{\overline{m}\left(c\right)\left(\overline{V}'\right)^2}{4\overline{F}\overline{k}}\left(\frac{d_1^2 k_1}{m_1\overline{d}^2} - \frac{d_2^2 k_2}{m_2\overline{d}^2}\right), \quad (3)$$

where $\overline{V}' = \overline{F}s$ is the mean fluctuation velocity; $\overline{F}$ is the mean fluctuation frequency calculated according to Ackermann–Shen's method [5]; $\overline{m}\left(c\right), \overline{k}$ are the average mass and coefficient of restitution at collisions of particles calculated for binary particulate mixture according to [4].

Thus, in order to predict the dynamics of particle distribution in a shear flow according to equation (1) we must determine coefficient K_s which is a part of the segregation kinetics equation

$$j_s = K_s \Delta, \tag{4}$$

where Δ is the driving force of the process.

Equation (4) parameters arc determined [2] on the basis of the following postulate: the parameters of the particle shear flow (fraction of solid volume, shear rate) being the same, the segregation flux is proportional to the degree of the local nonuniformity of the granular medium. Then, the kinetic parameter of the process K_s being constant, the segregation flux is evidently proportional to the degree of nonuniformity of the particulate medium, which is due to the addition of one component to another. That is why the driving force of the process is expressed in the form of the mixture variance. For a conventional binary mixture it follows that

$$j_s = K_s \rho_b c(1-c). \tag{5}$$

Thus, it would be logical to resume that the kinetic coefficient of segregation K_s is proportional to the degree of nonuniformity of the particle properties of the mixture (size, form, density, roughness, resilience, adhesion properties etc.). It is also evident that the degree of variation of particle properties will differently affect the intensity of segregation depending on the conditions determined by the flow hydrodynamics: the fraction of void volume $\varepsilon(y)$ and the shear rate du/dy. In connection with that in a general case of modeling the segregation dynamics we have to deal with extremely non-linear problems and solve all the difficulties resulting from them. The necessity of determining K_s by solving the inverse non-linear problem using experimental data on segregation dynamics, which are difficult to obtain, could lead to the situation when equation (1) would lose its scientific and practical value to a great extent.

In its nature kinetic coefficient K_s is a reflection of the influence extent of particle nonuniformity on the segregation intensity with due regard to the conditions of particle interactions in the flow. So it might seem reasonable to find an universal parameter of such nonuniformity and to use it to calculate the segregation coefficient. This parameter is determined in [2, 6] as the excess sum of moments of forces acting on the test particle relatively some instantaneous axis of the particle rotation

$$\Delta M = M - M_0, \tag{6}$$

where $M = M_G + M_F + M_C$ is the total momentum of the gravity, friction and impact forces acting on the test particle in the real flow, while M_0 is the analogous sum of moments acting on an average particle in a conventionally uniform medium.

The value of the nonuniformity parameter ΔM conventionally determines the intensity and the direction of segregation depending on the most important characteristics of the medium and the flow: particle size and density, the coefficients of friction and restitution at collisions, shear velocity and the fraction of void volume.

It is supposed that the segregation coefficient K_s for cohesionless particles close to a sphere in their shape is determined in [2] as a linear function of the nonuniformity parameter ΔM

$$K_s = K \Delta M, \tag{7}$$

where K is the coefficient of relative rate of segregation, which is equal to the displacement velocity of the mixture component per unit of the excess momentum ΔM.

In this case the excess momentum ΔM is the measure of particles nonuniformity for the conditions of their interaction in the flow. As a result the problem of segregation coefficient determination is sufficiently simplified, but it is still rather hard because it is necessary to obtain reliable experimental data on segregation dynamics $c(\tau, x, y)$. Besides, the accuracy of segregation coefficient determination by solving the reverse problem of segregation dynamics is reduced because of evident errors. These errors inevitably take place during the evaluation of the flux intensity of quasi-diffusional mixing and particle migration accompanying the segregation flux.

In this paper the method of direct experimental determination of segregation coefficient is proposed. The method is used for closer definition of segregation kinetics and study of the single large and small particle movement during rapid gravity flow of particulate solids.

2 Experimental unit and methods

In this paper section the method of direct determination of the segregation coefficient in the course of a rapid gravity flow of cohesionless spherical particles is described. The method proposed should be considered as a stationary mode method, because the primary data for determination of the kinetic characteristics of segregation process are obtained by measuring the velocity of the test particle displacement in the stationary conditions of the test particle interaction with the bulk particles of the particulate material flow.

It is obvious, that the stationary conditions of interaction are obtained in the shear flow of uniform particles, when there are no gradients of shear rate and solid phase concentration. In this case, when the movement dynamics of a single test particle differing in its properties from those of the flow particles is analyzed, it seems possible to neglect the quasi-diffusion and migration. Finally, in such a flow the test particle will move at a constant relative velocity, which will be equal in its value to segregation coefficient K_s in accordance with the segregation dynamics equation (1). In spite of the fact that this method seems to be simple in theory, the serious experimental difficulties arise when it is put into practice.

First of all at present the adequate forecasting the flow parameters faces often a lot of experimental and analytical problems [11–21]. Ackermann and Shen [15] used geometrical analysis of the shear flow microstructure and came to the conclusion that the gravity flow should be characterized by a considerable lateral mass transfer (quasi-diffusion) which must be taken into consideration for an adequate flow modelling.

Many authors have pointed that rapid shear flows are analogous to molecular gas dynamics. This allowed some of them to apply a well-elaborated kinetic gas theory for solving the problem of granular materials flow dynamics. So, Savage and Jeffrey [18], Jenkins and Savage [14], used the term granular temperature to estimate the intensity of chaotic movements and their influence on the flow dynamics.

On the other hand the considerable experimental difficulties complicate the rapid gravity flow study of granular materials. It is rightly pointed out by Savage [16], that rapid gravity flows of particles down a steep incline appear to be extremely complicated for an experimental study despite their apparent simplicity. The analysis of the results of such investigations of flows shows (Savage [16], Hutter and Sheiwiller [17]) that the main difficulties arise from the high sensitivity of gravity flows to an internal probe and the largest boundary effects impeding in the use of visual investigation methods At present, these difficulties promote the appearance of new alternative experimental methods, basing on the use of several penetrating radiations [22–24].

At present in widespread cases of practice and scientific research the computer tomograph measurements are used. Denes, Szepvolgy et al. [25] carried out computer tomograph measurement in a gravity flow of particles during hopper discharge. The modern X-ray computer tomograph "Siemens Somatom Plus" was used to observe the dynamics of the flow boundary change in the course of the discharge process. Unfortunately, in spite of the unique technique this investigation method has not been developed to determine the local microstructure parameters.

Dolgunin and Ukolov [2] suggested experimental and analytical method of determination of the velocity and concentration profiles in the rapid gravity flows of particulate solids on a rough chute.

The experimental part of the method consists in sending a granular material down a stationary incline and collecting the particles in a tray containing a number of cells (Fig. 1). In accordance with the method developed, the experimental data include the bed depth h at the discharge threshold in a steady flow, time t, the material distribution function $G(x_1)$ along a certain horizontal coordinate axis x_1 located at a vertical distance H from the discharge threshold and the bed inclination angle α (Fig. 1).

The equations connecting the profiles of velocity $u(y)$ and the fraction of void volume $\varepsilon(y)$ in the rapid gravity flows of particulate solids on a rough chute are formulated as follows

$$|\vec{u}| = \frac{x_1 - y\sin\alpha}{\cos\alpha\sqrt{(H + y\cos\alpha - (x_1 - y\sin\alpha)\mathrm{tg}\alpha)\frac{2}{g}}}; \tag{8}$$

$$u(y, x_1)\rho(1-\varepsilon(y)) = G(x_1)\,. \tag{9}$$

Using the hypothesis about the analogy between the parameters of the granular medium under the rapid shear and the corresponding parameters of dense gas the authors obtained equation of the granular medium state

$$P(y)\bar{\varepsilon}(y) = \chi'\left(\frac{du}{dy}\right)^2, \tag{10}$$

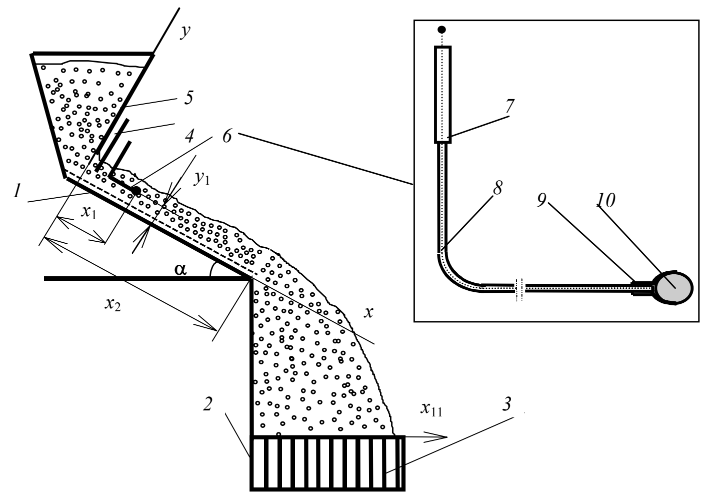

Fig. 1 Schematic of experimental unit:
1 – open channel; *2* – tray; *3* – cells; *4* – slide–valve; *5* – dosage devise; *6* – appliance for the test particle entry; *7* – tube; *8* – rope; *9* – clip; *10* – test particle

where $P(y) = \int\limits_{h-y}^{h} \rho^*(y) g \cos\alpha dy$ is the hydrostatic pressure analogue; $\bar{\varepsilon}(y) = \frac{1}{1-\varepsilon(y)} - \frac{1}{1-\varepsilon_o}$ is the bed dilatation, $\frac{du}{dy}$ is the shear rate, χ' is the equation coefficient of the granular medium state.

The equations (8)–(10) make up a closed system, relative to the functions $u(y)$, $y(x_1)$, $\varepsilon(y)$ and $P(y)$. The profiles of velocity and fraction of void volume are obtained solving the system of these equations by using the successive approximations.

It is noteworthy, that this method allows predicting the complex concentration profiles taking place in the rapid gravity flows having low bed heights. The method adequacy was indirectly corroborated in the course of repeated segregation dynamics modeling [2, 3] and directly checked by means of X-ray photography [26].

Besides the experimental problem is connected with the technical difficulties in providing the structural uniformity and constant shear rate along the whole shear flow in consequence of boundary effects. In the present paper we suggest to determine the segregation coefficient by measurement the velocity of test particle transverse displacement in the gravity flow of uniform bulk particles on a rough inclined chute. Our investigations [2, 3] and experimental results in [7] testify to the fact that the stationary conditions for the test particle interaction with the bulk particles are provided only in a certain area of the moving bed. It was found out that the relatively stable values of shear rate and fraction of void volume are observed in the central part of the bed, when particles move on a rough inclined chute in a steady sliding flow [8] and the bed is thick enough.

The method devised presupposes putting the test particle in the above mentioned flow part and determination its displacement velocity either in the direction of the bottom or in the direction of the free surface of the bed. The experimental device is very similar to the one described in Refs [2, 9].

The test unit (Fig. 1) consists of the rough inclined open channel of square cross-section. The rough bottom of the channel was installed at the angle of repose of the bulk materials to the horizon approximately. The roughness of the bottom of the chute equals half of the bulk particles diameter. There is a dosage device in the upper part of the chute and a horizontal tray under the discharge threshold at some distance below the slope. The tray is divided by some transverse partitions into cells. At a certain distance from the bottom of the chute between the discharge threshold and the dosage device there is an appliance for the test particle entry directly into the bed of the bulk particles (Fig. 1). It consists of a thin rigid tube with the end bent in the direction of the flow. On the bent end of the tube there is a clip, having a few plastic petals to grip the test particle. The clip connected with a flexible rope situated into the tube. The rope serves to force the test particle from the clip. The test particle is fixed into the clip and the flow particles move around it. When the rope is pushed the test particle is released and washed out by the flow of particles.

The technique of the experiment is as follows. First of all the coordinates x_1, y_1 (Fig. 1) of the test particle entry into the granular material flow are defined. To do this it is necessary to predetermine some thickness of the bed of the material being discharge. The thickness of 12 – 15 particle diameters is recommended as the first approximation. Then the profiles of velocity and fraction of void volume in the steady state of the flow are determined using the particle distribution function in cells of the tray and the interrelation function between bed dilatation and shear rate (10).

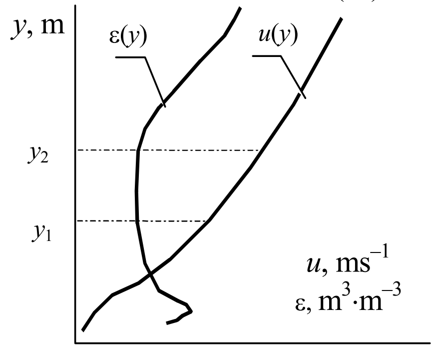

Fig. 2 Typical experimental profiles of velocity $u(y)$ and fraction of void volume $\varepsilon(y)$ in a steady sliding flow of particulate solids on a rough incline chute:
y_1, y_2 – initial and final coordinates of a large test particle

Fig. 2 shows the typical profiles of velocity and fraction of void volume in a rapid granular flow on a rough inclined chute in the mode of steady sliding flow. The profiles obtained help to determine the flow area boundaries which correspond to the conditions of the shear rate and fraction of void volume constancy ($du/dy \approx \text{const}$, $\varepsilon \approx \text{const}$). If the bed particles are uniform there are steady conditions for the interaction between bulk particles and the test particle and it is possible to neglect of its quasi–diffusion and migration. If thickness of this area is not big enough, increasing the dosage device capacity enlarges it. On the basis of the forecasts concerning the direction of the test particle displacement in this area of the flow we set its entry coordinate y_1. Analyzing the test particle properties we can choose the position of particle entry coordinate y_1 either at the low or at the top boundary of the area. For instance, if the test particle differs from the bulk particles only by its greater size then y_1 is situated at the low boundary of the area (Fig. 2). When it is difficult to forecast the direction of the test particle displacement the entry coordinate y_1 is situated in the middle of the flow area.

To determine coordinate x_1, which shows the distance from the point of test particle entry to the point where the bulk material is dosed (Fig. 1), the moment of coming of the steady state of the particulate material flow is determined. It is done by measurement the profiles of velocity and fraction of solid volume either along the bed or at the discharge threshold, when the dosage device is gradually moved towards the threshold. The moment of steady state coming is defined, the dosage device is placed at such a distance from the discharge threshold which ensures the entry of the test particle into the stationary flow and its movement in the flow for a certain period of time.

At the next investigation stage the coordinate y_1 of the test particle entry is defined more accurately. This operation is carried out by finding the mean statistical value of the results of repeated determinations of y_2 coordinate at the discharge threshold for a marked particle having properties analogous to those for the bulk particles. It is necessary in order to correct possible inaccuracy of the entry coordinate y_1 determination, which could be as a consequence of the turbulence, producing by the particle entry device. The analysis of the investigation results shows that the difference between the directly measured and the corrected value of y_1 is about $1 \cdot 10^{-3}$ m.

The test particle is fixed in the channel by the entry device at the point of coordinates x_1, y_1 and the bulk particles are dosed. Primary the bulk material is collected in the buffer vessel and when the steady flows are achieved the test particle is put into the bed by means of the entry device. Simultaneously the buffer vessel is taken away and flow particles are collected in the tray cells. The test particle coordinate y_2 at the channel exit is determined using the profiles of velocity and fraction of void volume in the flow at the discharge threshold and taking into account the fact which of the tray cells the test particle gets into. In order to get a more reliable value of the particle transverse velocity, its exit coordinate y_2 is found by means of determination the mean value of all the results got in the numerously repeated experiment.

Presuming that within the area with constant du/dy and ε the test particle moves in direction y at a constant velocity we arrive at the conclusion that its movement along coordinate x of the main flow is uniformly accelerated. The acceleration of the test particle movement in direction x from x_1 to x_2 equals

$$a = \left(u\left(y_2\right) - u\left(y_1\right)\right)\tau^{-1}, \tag{11}$$

where τ is the test particle movement time in the bulk particle flow.

The distance traveled by the test particle in the direction of the slope is determined in the following way

$$x_2 - x_1 = u\left(y_1\right)\tau + 0,5a\tau^2. \tag{12}$$

Equations (11) and (12) allow us to determine the test particle movement time in the bulk particle flow as well as component u_y of its velocity in direction y

$$u_y = 0,5\left(y_2 - y_1\right)\left(u\left(y_1\right) + u\left(y_2\right)\right)\left(x_2 - x_1\right)^{-1}. \tag{13}$$

In its formal sense coefficient K_s according to the kinetic model of hydromechanical segregation (5) is equivalent to velocity of the transversal displacement of test particles during segregation in a shear flow of particulate solids, i.e. $u_y \approx K_s$.

Then, supposing linear relationship (7) between K_s and the nonuniformity parameter ΔM we can formulate

$$K = u_y\left(\Delta M\right)^{-1}. \tag{14}$$

It seems to be that coefficient K, which determines the relative rate of segregation, has rather important forecasting properties, allowing to predict the velocities of relative displacement both of single particles and mixtures components during rapid shear flow of particulate solids. However, it needs to be checked in terms of adequacy of the mechanism of hydromechanical separation used and its modeling.

3 Segregation of single particles in a rapid shear flow

To check the hypotheses made about a linear dependence between kinetic parameters in Eqs. (7) and (14) these segregation characteristics were experimentally investigated. The coefficient of segregation K_s and the coefficient of relative rate of segregation K were determined as functions of the diameter of test particles. The ceramic balls, ammophos granules and glass beads were used as modelling bulk materials in the experiment. The basic physical and mechanical properties of modelling bulk materials are presented in table 1.

Table 1

Modeling bulk materials and their physical and mechanical properties

Material	Particle size, $d_b 10^3$, m	Particle density, ρ, kg·m^{-3}	Bulk density, ρ^{-3}, kg·m^{-3}	Angle of repose, degree	Restitution coefficient, k	Friction coefficient
Ceramic balls	6,6	2086	1251	36	0,82	0,85
Glass beads	+2,25…2,5	2500	1500	26	0,9	0,7
Glass beads	+3,25…3,5	2500	1500	26	0,9	0,7
Ammophos granules	+3,0…3,25	1640	984	32	0,5	0,75

During this experiment the velocities of the transverse displacement of single large and small test particles were defined in the rapid gravity flow of uniform particles on a rough chute. We used the experimental unit and methods described in Section 2 of this paper. According to this method the transverse velocity of a test particle is determined on a rough chute in the flow area having constant values of the shear rate and the fraction of void volume. This velocity in accordance with its physical sense is adequate to coefficient K_S, when the test component concentration is closed to zero, i.e. $c \to 0$. In this case the stable values of flow parameters allow easy to calculate the nonuniformity parameter ΔM [2, 6]. The physical and mechanical properties of modelling bulk materials given in Table 1 were used during ΔM calculation.

The corresponding value of the coefficient of relative rate of segregation is defined as the ratio of K_S to ΔM, i.e.

$$K = K_S \left(\Delta M\right)^{-1}. \tag{15}$$

Fig. 3 shows the profiles of velocity and fraction of void volume in rapid gravity flow of bulk materials presented in Table 1. The profiles were obtained according to the method of segregation coefficient determination described in the previous part of this paper. These profiles allow to allocate the areas situated in the central part of the flows and having constant values of shear rate du/dy and the faction of void volume. These flow areas were used to determine the transversal velocity of test particles differing in size from bulk particles.

Fig. 4 shows the kinetic parameter K_S, which is equivalent to the transversal velocity of test particles, and nonuniformity parameter ΔM as dependencies of the test particle diameter d_t for corresponding bulk materials. The dependencies obtained cross each other in the point lying on the abscissa axis and which coordinate is equal to the mean particle diameter of particulate medium. The K_S and ΔM values on the left of this point were obtained when the small test particles ($d_t < d_b$) moved towards the flow bottom whereas the values on the right of this point were defined for the case when test particle diameters were greater than the mean diameter of the bulk particles ($d_t > d_b$) and when the test particles moved towards the free surface of the flow.

Processing the data presented in Fig. 4 with due regard to Eq. (14) we obtained the values of the kinetic coefficient K which determines the relative rate of segregation. Fig. 5 shows coefficient K as a dependence of test particle diameter for different modelling materials.

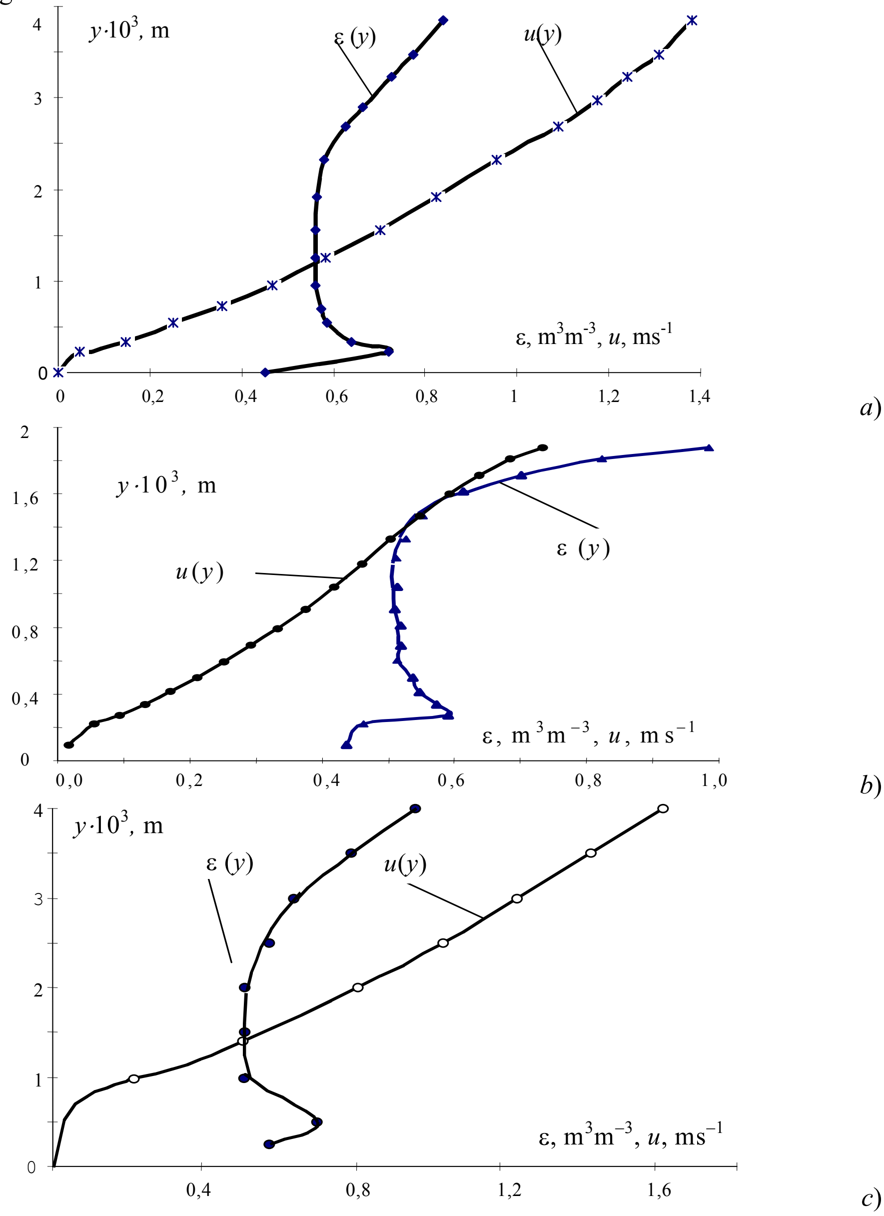

Fig. 3 Profiles of velocity $u(y)$ and fraction of void volume $s(y)$ in rapid gravity flows of:
a – ceramic balls of $6,6 \cdot 10^{-3}$ m diameter; b – glass beads of fraction $(+3,25-3,5)\cdot 10^{-3}$ m ;
c – ammophos granules of fraction $(+3,0-3,25)\cdot 10^{-3}$ m

The results presented in Fig. 5 allow indicating the fact that in the investigated range of particle size ratio coefficient K may be regarded as a kinetic constant for a certain kind of material. This fact reflects the linear dependence between the segregation rate of a single test particle K_s and its nonuniformity parameter ΔM expressed in terms of the conventionally uniform particles of a rapid gravity flow. The properties found out for parameters K and ΔM meet the traditional requirements of the kinetic coefficient and the driving force of a technological process respectively.

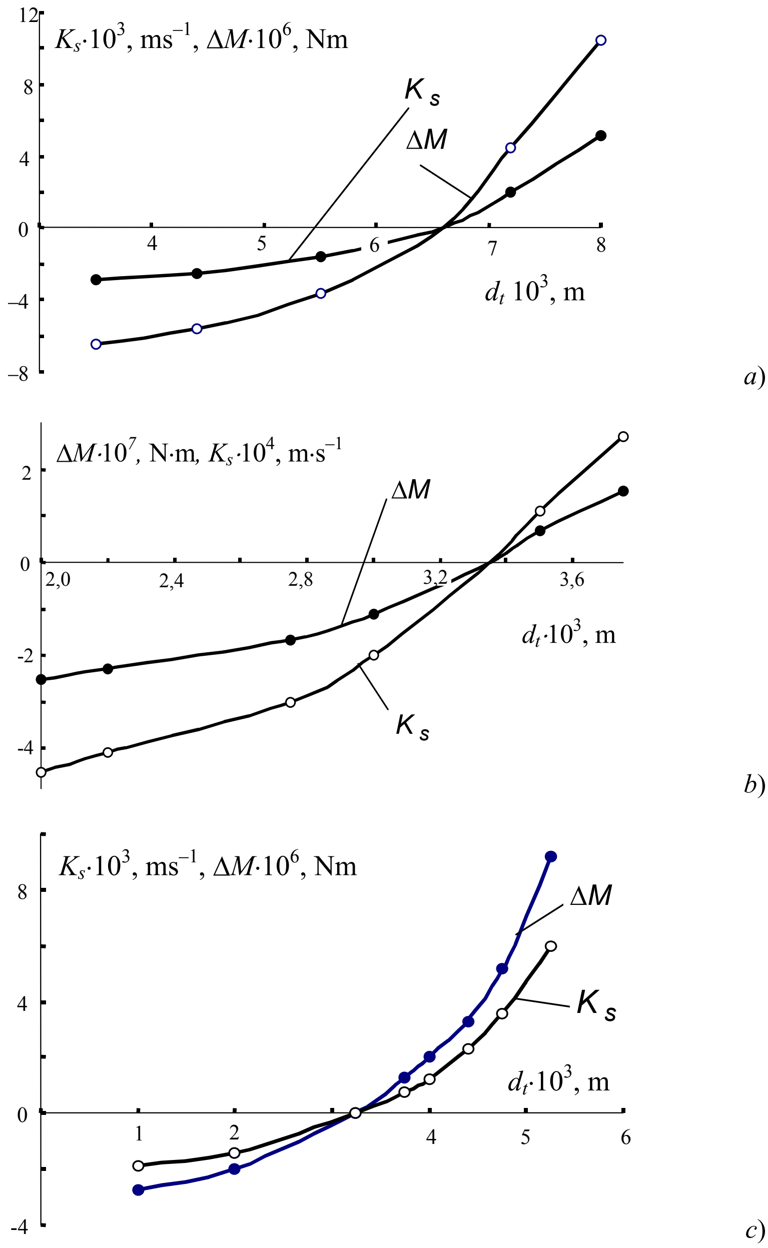

Fig. 4 Relative velocity K_s and nonuniformity parameter ΔM as dependencies of test particle diameter in rapid gravity flows of:

a – ceramic balls of $6,6 \cdot 10^{-3}$ m diameter; *b* – glass beads of fraction $(+3,25-3,5)\cdot 10^{-3}$ m ;

c – ammophos granules of fraction $(+3,0-3,25)\cdot 10^{-3}$ m

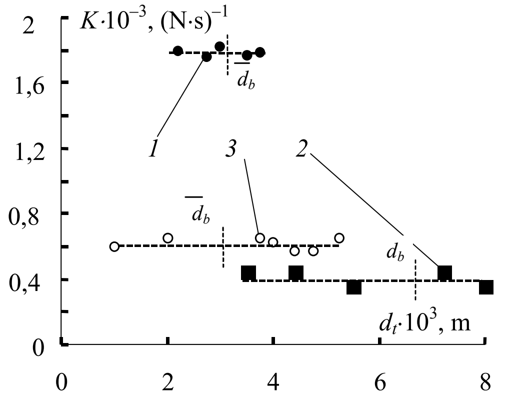

Fig. 5 Coefficients K of segregation kinetics as dependence of the test particle diameter d_t in rapid gravity flows of:

1 – glass beads ($d_b = (3,25...3,5)\cdot 10^{-3}$ m); *2* – ceramic balls ($d_b = 6,6\cdot 10^{-3}$ m);

3 – ammophos granules ($d_b = (3,0...3,25)\cdot 10^{-3}$ m)

In our paper [2] the postulate was propounded and substantiated to use the variance of a conventionally binary particulate medium $c(1-c)$ as the driving force of segregation. Then the kinetic coefficient K was determined by the solving of an inverse problem of segregation dynamics basing on the experimental concentration profiles of test component distribution in a rapid shear flow. The obvious drawbacks of this method are the difficulties to obtain the reliable experimental profiles of velocity, fraction of void volume and test particle distribution. Besides the accuracy of the K coefficient determination according to this method may be very low in consequence of errors which appear when the migration and quasidiffusional fluxes are taken into account with due regard to Eq. (1).

It is noteworthy that a basis appears to check the hypotheses of the driving force expression by means of the suggested method of direct experimental determination of segregation coefficient and the segregation kinetic law formulation. In the present paper the research on these problems is carried out.

To test additionally the parameters ΔM and K as the kinetic characteristics of segregation we have investigated the segregation kinetics in two different gravity flows of beads differing from each other in flow parameters and particle sizes. The experiments were carried out by the measurement of velocity $u_y = K_s$ of the relative transverse displacement of single test particles in steady rapid gravity flows on the rough chute (Fig. 1). In the first experiment the K_s value had been determined when the test particle of diameter 3,5 mm moved towards the free surface in the rapid gravity flow of particles having diameters +2,25 – 2,5 mm. In the second experiment the velocity K_s had been defined for the test particle of diameter 2,5 mm was moving towards the bottom in the rapid gravity flow of particles of fraction +3,25 – 3,5 mm. According to the significantly different behaviour of the test particles in these experiments the positive value of K_s in the first experiment and the negative – in the second one were obtained.

At the following stage taking into consideration the different values of shear velocity, fraction of void volume, physical properties and particle sizes in experiments 1 and 2 we calculated the driving force of segregation as the excess sum of the momenta of

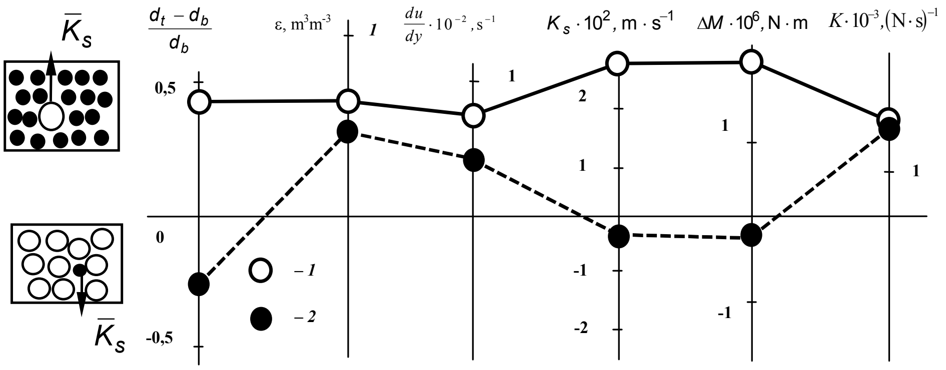

Fig. 6 Schematic of particulate medium properties, flow parameters, experimental and calculated characteristics during glass beads gravity flow on a rough chute in experiments *1* and *2*:

1 – $d_t = 3{,}5$ mm, $d_b = +2{,}25 - 2{,}5$ mm ; *2* – $d_t = 2{,}5$ mm, $d_b = +3{,}25 - 3{,}5$ mm

gravity, friction and impact forces ΔM, acting on the test particle. And at last, the kinetic coefficient of segregation K was determined as the ratio of K_s to ΔM for each of the experiments.

Fig. 6 shows schematic of conditions of both the above mentioned experiments 1 and 2 (particle ratio, shear rate du/dy, fraction of void volume ε), the measured transverse velocity of the test particles K_s, the calculated values of segregation driving force ΔM and the coefficient of relative rate of segregation K. The data shown in Fig. 6 testify to the fact, that in spite of the reverse segregation effects and the significant difference in compositions of these media and their flow parameters in particulate media 1 and 2 the observed segregation effects are characterised by the same value of the coefficient of segregation kinetics K.

Thus the investigation carried out allows to establish that the coefficient of relative rate of segregation K and nonuniformity parameter ΔM meet the traditional requirements of the kinetic coefficient and the process driving force respectively: segregation rate is proportional to K and ΔM, K may be regarded as a constant and ΔM reflects the influence of particle properties and flow parameters on segregation intensity. However, the investigation carried out is based only on the kinetic analysis of the relative velocity of movement of single test particles in conventionally uniform particulate media. That is why, it is important to note that the conclusions made may be completely concern only to the particulate media having the low concentration of test particles, i.e. when $c \to 0$, $c \to 1$. Obviously the additional investigation is necessary to extend this approach for particulate mixtures, when $0 < c < 1$.

4 Segregation kinetics in a rapid gravity flow of particulate mixtures

The segregation driving force ΔM is the measure of the medium nonuniformity, which defines the deviation of the real medium properties from the properties of a conventionally uniform medium under the same flow conditions.

Thereby, the most principal problem on the way of driving force expression for a particulate mixture is the determination of the parameters of a conventionally uniform medium. The two basic versions were analysed in Ref. [10] to solve this problem. The first one assumes that the conventionally uniform medium consists only of the bulk particles having concentration $(1-c)$ in a particulate mixture. The second version assumes that the uniform medium consists of particles, having the properties, which are determined as a characteristic ones for the particulate mixture on the whole.

The use of the second version is being preferable in spite of its more arbitrary nature at least due to the following circumstances. The first one is the fact, that the parameters of a rapid shear flow (shear rate, fraction of void volume, fluctuation velocity) are already calculated by assumption of uniform properties of particles with due regard to the typical interaction of real particles. The second circumstance consists in the fact, that segregation is a total effect appearing as a final result of particle interactions at various their combinations. Besides, it can be maintained, that a particle of such conventionally uniform medium will not segregate if it will be put into the real particulate medium under certain flow conditions. Thus, the parameters of such hypothetical uniform medium can be considered as the equilibrium conditions of segregation process. Then the idea arises, that the driving force of segregation may be expressed as a difference between the dynamic parameters, when the first of which describes the particle interactions in a hypothetical uniform medium and the second one is being the analogous parameter obtained when the test particle interacts with conventionally uniform particles. It is obvious that these two versions of segregation driving force expression have to be analysed in terms of their adequacy.

The versions mentioned above are analysed in the present paper both taking into consideration the earlier stated postulate [2] that the segregation flux is proportional to the concentration variance of a conventionally binary particulate mixture $c(1-c)$ and without taking into account this postulate, when the segregation flux is proportional to the test component concentration c. However, if we take into account, that parameter ΔM in the first version of its determination does not depend on the test component concentration, we come to a conclusion about the necessity always to apply in this case the postulate when the segregation flux is proportional to $c(1-c)$. Otherwise, the maximum value of segregation flux will take place at concentration $c \to 1$ that obviously conflicts with physical sense. In this connection the first version of parameter ΔM determination is analyzed only considering multiplier $c(1-c)$, and the second one is analyzed both with the multiplier $c(1-c)$ and with multiplier c. The analysis is carried out by a mathematical simulation of segregation dynamics (concentration profiles of a test component) in gravity flows of beads and ceramic balls on the rough chute.

The concentration profiles were calculated on the basis of Eq. (1) which was integrated numerically for the initial condition $c(0, x, y) = c_0$, where c_0 is the mean concentration of a test component. Boundary conditions were formulated assuming the absence of a transverse flux at the free surface and the bottom of the bed. All the kinetic coefficients in Eq.(1) excepting K_s were calculated analytically using traditional physical and mechanical characteristics of particles.

Coefficient K_s was calculated according to Eq. (7) using the experimentally measured values of the kinetic coefficient $K = \text{const}$ obtained in the previous section (Fig. 5).

The concentration profiles of the test component distribution in a gravity flow on the rough chute are shown in Fig. 7. The particle sizes of binary mixtures and test particle concentrations are choosen to investigate the segregation kinetics behaviour according to the physical mechanism of hydromechanical separation in a wide range of change of the segregation driving force and to test the forecasting properties of various versions of the kinetics equation. The results of various versions of segregation kinetics simulation are shown also in Fig. 7. To estimate the adequacy of these simulations the corresponding experimental concentration profiles are shown in Fig. 7. The experimental profiles were obtained using the experimental and analytical method described in Ref. [2].

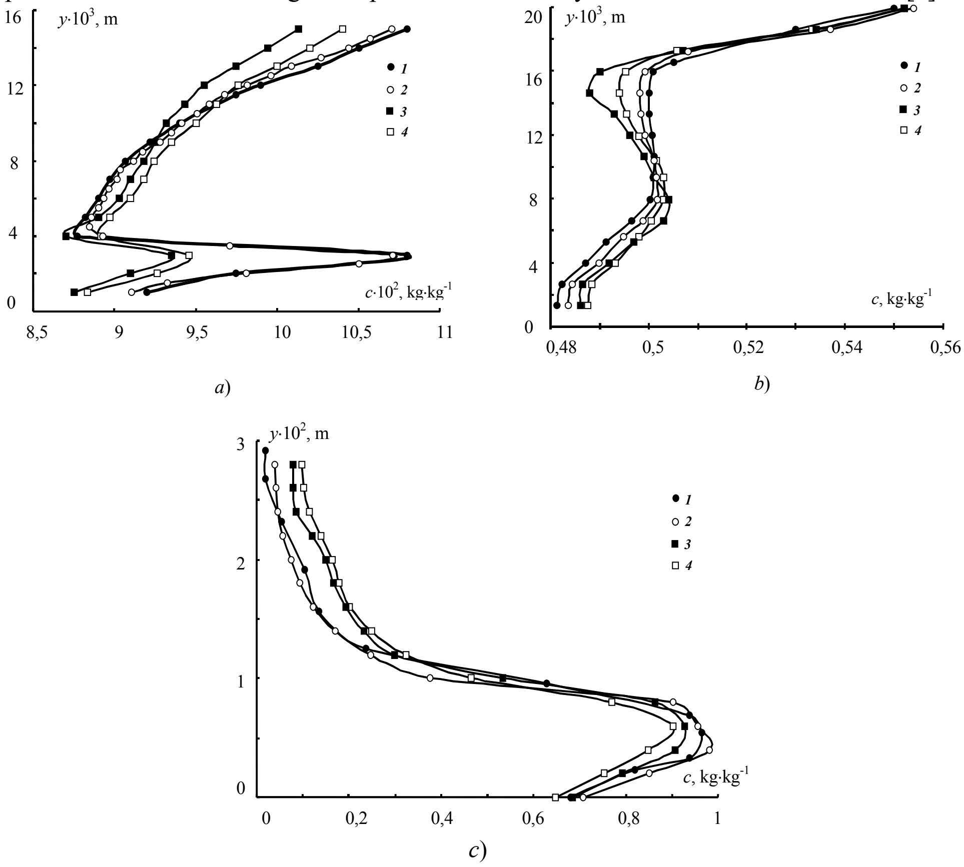

Fig. 7 Comparison of experimental (*1*) and calculated (*2*, *3*, *4*) profiles of concentration during gravity flow of particulate mixtures:

a – beads consisting of fractions (3,6 – 3,75) 10^{-3} m – 10 %, (3,25:3,5) 10^{-3} m – 90 %;
b – beads consisting of fractions (3,0 – 3,25)·10^{-3} m – 50 %, (3,5 – 3,6)·10^{-3} m – 50 %;
c – ceramic balls consisting of fractions 6,6·10^{-3} m – 90 %, 4,4·10^{-3} m – 10 %
at various versions of the driving force Δ expression:
2 – $\Delta = \Delta M$; *3* – $\Delta = c(1-c)\Delta M$ (version 1); *4* – $\Delta = c(1-c)\Delta M$ (version 2)

The analysis of the results shown in Fig. 7 allows making a conclusion that in all the cases of numerical simulation only the results, basing on the version that the segregation driving force is adequate to the nonuniformity parameter ΔM are quite satisfactory. In this case the equation of segregation kinetics is formulated as follows

$$j_s = K\rho_b \Delta M c = K\rho_b \left(M - M_0\right) c . \tag{12}$$

Thus ΔM is determined according to the second version of the driving force expression, i.e. in the assumption, that the uniform medium consists of particles, which parameters are characteristic ones for the whole medium. It is necessary to note, that this version is theoretically the most reasonable among all the versions analysed in the present paper.

Really, only this version takes into account all the possible combinations of particles interactions taking place in a nonuniform particulate medium. According to this fact, there is no necessity to postulate in addition any dependence of the segregation driving force from the test particle concentration. In this case the nonuniformity parameter

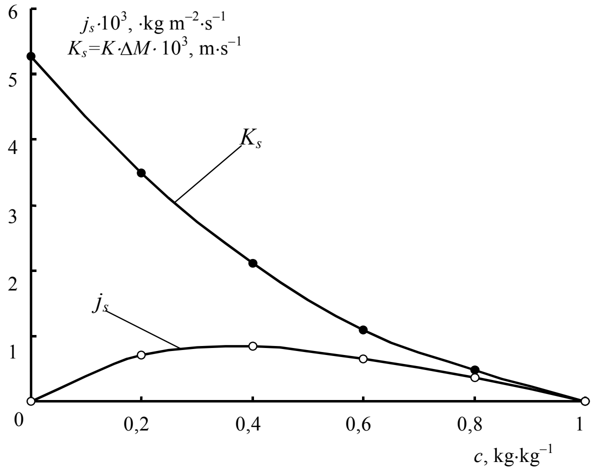

Fig. 8 Velocity K_s of segregation flux of test particles and segregation flux value j_s as dependencies of the test component concentration during rapid gravity flow of ceramic balls of diameters 4,4 and 6,6 mm (calculated at $du/dy = 57,\ \mathrm{s}^{-1}$ **;** $\varepsilon = 0,5,\ \mathrm{m}^3 \cdot \mathrm{m}^{-3}$ **;** $d_t = 4,4$ mm)

ΔM , being a function of the test particle concentration, have already reflected a dependence of segregation rate from concentration. This conclusion is corroborated by the calculation results presented in Fig. 8.

Fig. 8 shows the variation of the transversal velocity of segregating test particles at different their concentrations in the binary mixture of ceramic balls during rapid gravity flow without any gradients of shear rate and the fraction of void volume. The velocity of the segregation flux was calculated as $u_y = K_s = K\Delta M(c)$, where $K = 4,1 \cdot 10^2\ N^{-1} s^{-1}$ according to the data shown in Fig. 5.

The results obtained show that the test component velocity is equal to its maximum value when the test component concentration $c \to 0$. It is obvious, that in this case the test component velocity is closed to the percolation velocity of a single test particle. If the test particle concentration increases the segregation velocity of these particles decreases and it approaches to zero when $c \to 1$.

Besides, Fig. 8 shows the segregation flux as a dependence of the test particle concentration. This result allows to observe the fact that segregation flux depends almost linearly on the test component concentration when concentration approaches to its limit values, i. e. when $c \to 0$ and $c \to 1$. On the other hand we can observe that the segregation flux is equal to zero in the uniform media ($c = 0$; $c = 1$).

Thus the kinetic features of segregation observed do not contradict the physical nature of this phenomenon.

Then taking into account kinetic Eq. (15) corrected we can rewrite the general equation of segregation dynamics (1) as follows

$$\frac{\partial(c\rho_b)}{\partial\tau} = -\frac{\partial(uc\rho_b)}{\partial x} + \frac{\partial}{\partial y}\left[\rho_b\left(D_{dif}\frac{\partial c}{\partial y} - D_m\frac{\partial \ln s}{\partial y}c - K\Delta Mc\right)\right]. \qquad (16)$$

It is noteworthy that all the kinetic characteristics of this equation excepting coefficient K are calculated analytically. The coefficient of segregation kinetics K is determined experimentally as the kinetic constant for a certain kind of particulate medium.

5 Conclusions

The method of direct determination of the segregation coefficient in the course of rapid gravity flow of cohesionless spherical particles is proposed. The primary data for the determination of the kinetic characteristic of the segregation process are obtained by measuring the relative velocity of a test single particle displacement. The nonuniformity parameter determined as the total excess momentum of gravity, friction and impact momenta forces acting on the test particle is used to generalise the investigation results on segregation kinetics. As a result, the unified kinetic constant is determined for a certain kind of material.

The suggested method of segregation coefficient determination was used to investigate the kinetic behaviour of segregation according to the mechanism of hydromechanical separation in a rapid shear flow. The different formulation versions of segregation driving force were analysed in terms of their adequacy. It was found out that the constant obtained and the nonuniformity parameter used meet the traditional requirements for the kinetic coefficient and the driving force of a technological process respectively. Thereby the kinetic coefficient of segregation may be considered as an universal kinetic constant for a certain kind of material to forecast both the velocity of a relative displacement of single large or small particles in uniform particulate media and the segregation dynamics in gravity flows of particulate solids of various particle sizes and various flow conditions.

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Кинетика сегрегации частиц в быстрых гравитационных потоках

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Ключевые слова: кинетика сегрегации; зернистый материал; быстрый гравитационный поток.

Аннотация: Проанализирована проблема описания кинетики сегрегации в быстрым гравитационном потоке зернистого материала. Предложен экспериментально–аналитический метод определения коэффициента сегрегации в быстром гравитационном потоке частиц. Этот метод используется для анализа кинетики сегрегации в быстром гравитационном потоке и для уточнения уравнения сегрегации, предложенного ранее. Исследованы прогностические свойства предложенной модели кинетики сегрегации. Модель использована для расчета динамики сегрегации зернистых смесей и скоростей поперечного перемещения одиночных крупных или мелких частиц в быстрых гравитационных потоках зернистых материалов.

Kinetik der Segregationsteilchen in den schnellen Gravitationsströmen

Zusammenfassung: Es ist das Problem der Beschreibung der Segregationskinetik im schnellen Gravitationsstrom des Körnerstoffes analysiert. Es ist die experimentell-analytische Methode der Bestimmung des Segregationskoeffizientes im schnellen Gravitationsstrom der Teilchen vorgeschlagen. Diese Methode wird für die Analyse der Segregationskinetik im schnellen Gravitationsstrom und für die Präzisierung der Segregationsgleichung benutzt. Es sind die Prognoseeigenschaften des vorgeschlagenen Modells der Segregationskinetik untersucht. Das Modell ist für die Berechnung der Segregationsdynamik der Körnergemische und der Geschwindigkeit der Querbewegung der einzelnen großen und kleinen Teilchen in den schnellen Gravitationsströmen der Körnerstoffe angewandt.

Cinétique de la ségrégation des particules dans de vites courants de gravitation

Résumé: Est analysé le problème de la description de la cinétique de la ségrégation des particules dans de vites courants de gravitation du matériel granulé. Est

proposée la méthode analytique expérimentale pour la définition du coefficient de la ségrégation dans le vite courant des particules. Cette méthode est utilisée pour l'analyse de la cinétique de la ségrégation dans le vite courant de gravitation ainsi que pour la précision de l'équation de la ségrégation proposée auparavant. Sont étudiées les particularités de prévision du modèle proposé pour la cinétique de la ségrégation. Ce modèle est utilisé pour le calcul de la dynamique de la ségrégation des mélanges granulés et des vitesses du déplacement des grandes et petites particules unitaires dans de vites courants de gravitation des matériaux granulés.