

**MATHEMATICAL MODEL OF ELECTRONIC OPTICAL MOIRE  
PATTERN OF MAGNETIC FIELD ON ELECTRONIC  
EQUIPMENT ELEMENT DEFECTS**

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**Key words and phrases:** electromagnetic field; field distortion; moire.

**Abstract:** Problems of interconnection between electronic equipment element performance and generated electromagnetic fields are viewed in the article. It is pointed out that concentration of electromagnetic energy near defects may result in electronic equipment failure. Techniques of observation and measuring of magnetic fields near defects on flat conductors are discussed and methods of their calculation with the help of electron optical moire are offered in the article.

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Performance of electrical equipment elements is closely connected with generated electromagnetic fields. There are usually some kinds of defects on thin flat conductors which appear in the process of production and operation and further define their electromagnetic compatibility. Concentration of electromagnetic energy near these defects may result in electronic equipment failure. That is why monitoring and measuring of magnetic fields near defects appears to be an actual task. Let us view some investigation results of these fields observation near defects on flat conductors and present methods of their calculation with the help of electron optical moire.

As model sample there was used cuprum plate with central aperture (80×80×1) mm, placed in the column of electronograph EG–100A so as to enable the cathode beam to slide down the surface maximally close to the defect (Fig. 1).

Magnetic field excitation in the plate was initiated by constant voltage  $U$  commutated by contacts of polarized relay. Power supply of relay winding was carried out from audio-frequency oscillator with frequency of 32 Gertz and controlled by an oscillograph. Moire pattern was obtained when two views were put together; that of undistorted grid and grid distorted due to Lorentz power appearing in magnetic field (Fig. 2).

For quantity analysis of magnetic field near defect by amount of cathode beam displacement on electronograph screen let us use the equation of charged particle motion on axis  $x$

$$\frac{d^2x}{dt^2} = \frac{e}{m} v_0 \mu_0 H_x \quad (1)$$

or

$$v_x = \frac{e}{m} \mu_0 v_0 \int H_x(z) dz, \quad (2)$$

where  $v_x$  – velocity of electron escape from the active field region;  $v_0$  – particle acceleration velocity;  $H_z$  – magnetic field near aperture;  $e, m$  – electron charge and mass;  $\mu_0$  – magnetic constant.

Finally we obtain

$$x = \frac{e}{m} \mu_0 \frac{L}{v_0} \int H_x(y) dy, \quad (3)$$

where  $v_0 = \sqrt{\frac{2eU}{m}}$ ;  $dz = v_0 dt$ ;  $x = v_x \frac{L}{v_0}$ ;  $U$  – accelerating voltage of 40 kV.

Visible on the screen beam deflection in a magnetic field can be calculated by the equation

$$x' = v_x \frac{L}{v_0} = \frac{e}{m} \mu_0 \frac{L}{v_0} \int H_x(y) dy, \quad (4)$$

where  $L$  – distance between the object and the screen equal to 0,4 m.

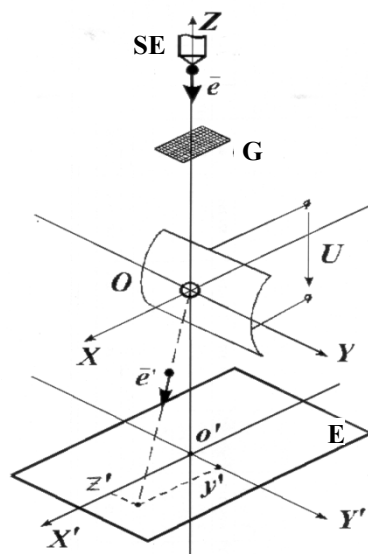
Let us specify magnetic field distribution on axis  $x$  which approximates rather precisely integration. Function describing bell distribution is used more often when flat magnetic lens are calculated [1]

$$H_x(y) = \frac{H_{xm}}{1 + \left(\frac{y}{5d}\right)^2}, \quad (5)$$

where  $d$  – aperture diameter.

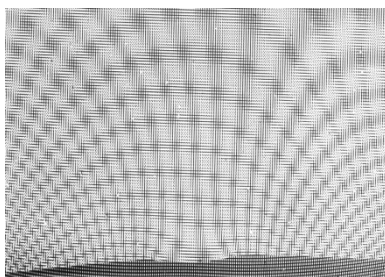
After data substitution the calculated ratio between deflection  $x'$  and  $H_x(y)$  is

$$x' = 3,5 \cdot 10^{-6} H_{x \max}. \quad (6)$$

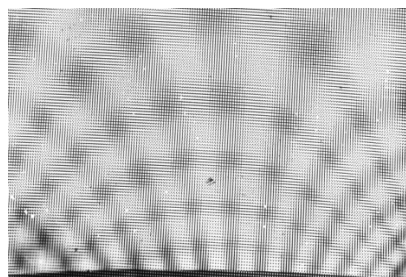


**Fig. 1. Experimental model (sample with central aperture):**

SE – source of electrons; G – grid; object (bent plate with aperture); E – electronoptical screen;  $\bar{e}$  – cathode beam;  $U$  – source potential



a)



b)

**Fig. 2. Electron-optical moire patterns of magnetic fields:**  
a – plates without aperture; b – plates with aperture,  $I = 5 A$

Beam deflection can be easily measured by moire patter, e.g.

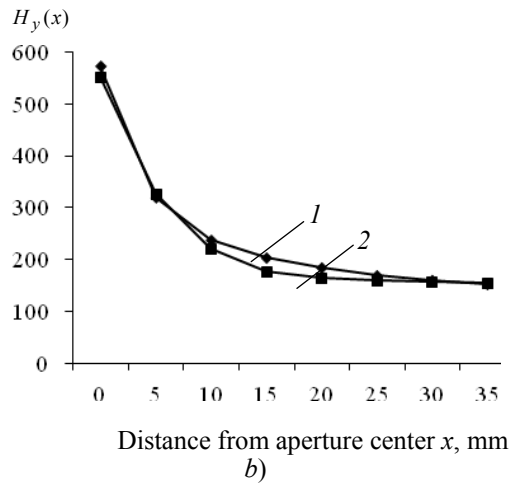
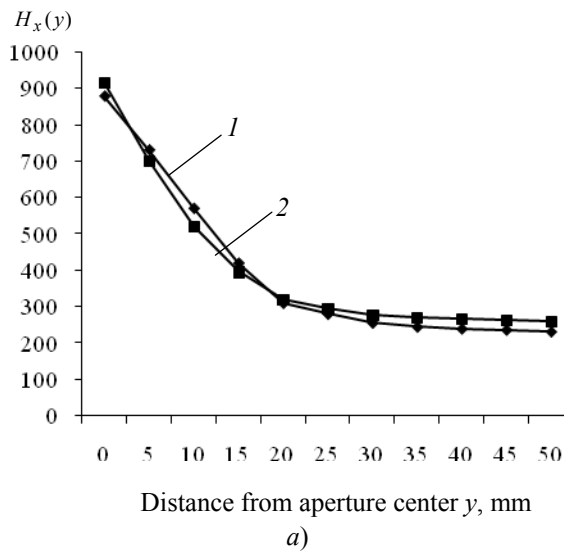
$$x' = km = 6 \cdot 0,5 \cdot 10^{-3} \text{ м}, \quad (7)$$

where  $k$  – equipotential number;  $m = 0,5 \cdot 10^{-3}$  m – the size of amplified grid cell picture.  
Magnetic field strength is found from the expression (6)

$$H_{xm} = \frac{x'}{3,5 \cdot 10^{-6}}. \quad (8)$$

Obtained data are compared with theoretical ones get by the formula

$$H_x(x, y) = \frac{I}{\pi R} \left[ \frac{1}{\sqrt{\left(1 + \frac{y}{R}\right)^2 + \left(\frac{x}{R}\right)^2}} k(\varphi) \right] + \left[ \frac{1 - \left(\frac{y}{R}\right)^2 - \left(\frac{x}{R}\right)^2}{1 - \left(\frac{y}{R}\right)^2 + \left(\frac{x}{R}\right)^2} E(\varphi) \right], \quad (9)$$



**Fig. 2. Distribution of axial (a) and radial (b) components of magnetic field in direct current:**  
1 – experimental; 2 – theoretical

where  $I$  – electrical current on the plate;  $R$  – aperture radius;  $k(\varphi)$ ,  $E(\varphi)$  – elliptic integrals of first and second type [2].

Equipotentials obtained by lines of vertical grid coincidence and characterizing radial magnetic field component are treated in similar way. In this case deflection on axis  $y$  can be calculated from the formula

$$y' = v_y \frac{L}{v_0} = \frac{e}{m} \mu_0 \frac{L}{v_0} \int H_y(x) dx. \quad (10)$$

After substituting of numerical data and bell distribution when  $d = 0,5 \cdot 10^{-6}$  m in equation (10) and integrating it we obtain the calculated ratio

$$H_{ym} = \frac{y'}{5 \cdot 10^{-6}}. \quad (11)$$

Theoretical and experimental dependence of axial magnetic field strength  $H_x(y)$  of a flat conductor with aperture when voltage is constant are shown on Fig. 2, a.

Data obtained on radial magnetic field component  $H_y(x)$  are treated in a similar way.

It follows from the comparison of theoretical and experimental data that divergency is not more than 5 % of the whole range, which proves the moire pattern adequacy to real heterogeneous magnetic field of a flat conductor.

It must be stressed that in all experiments the aperture in the plate was characterized by maximal moire pattern distortion.

Suggested technique of electron-optical moire may be used not only for magnetic field topography visualization but also for defects detection due to maximal moire pattern deflection.

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### Математическая модель электронно-оптической муаровой картины магнитного поля на дефектах элементов электрооборудования

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**Ключевые слова и фразы:** искажение полей; муар; электромагнитное поле.

**Аннотация:** Работа элементов электрооборудования тесно связана с генерируемыми электромагнитными полями. В тонких полосковых проводниках

всегда имеются монтажные, технические, геометрические и структурные дефекты, которые рождаются в процессе их производства и эксплуатации и определяют их электромагнитную совместимость. Концентрация электромагнитной энергии вокруг этих дефектов может привести к отказу электрооборудования, поэтому наблюдение и измерение магнитных полей вокруг дефектов, которые локализуются в малых объемах, является актуальной задачей. Приведены некоторые исследования по наблюдению этих полей вокруг дефектов на плоских проводниках и представлен механизм расчета с помощью электронно-оптического муара.

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### **Matematisches Modell des elektronenoptischen Moire-Bildes des magnetischen Feldes auf den Defekten der Elemente der elektrischen Ausrüstung**

**Zusammenfassung:** Die Arbeit der Elemente der elektrischen Ausrüstung ist mit den erzeugten elektromagnetischen Feldern eng verbunden. In den feinen Streifenleiter gibt es immer die montagen, technischen, geometrischen und strukturellen Defekte, die im Laufe ihrer Produktion und der Ausbeutung entstehen. Die Konzentration der elektromagnetischen Energie um diese Defekte kann zum Versagen der elektrischen Ausrüstung bringen. Die Beobachtung und die Messung der magnetischen Felder um Defekte, die in den kleinen Umfängen lokalisiert werden, ist eine aktuelle Aufgabe. Wir werden einige Forschungen nach der Beobachtung dieser Felder um die Defekte auf den flachen Leitern anführen und wir werden den Mechanismus der Berechnung mit Hilfe des elektronenoptischen Moires darstellen.

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### **Modèle mathématique de l'image de moire électronique et optique du champ magnétique sur les défauts des éléments de l'équipement électrique**

**Résumé:** Le fonctionnement des éléments de l'équipement électrique est étroitement lié aux champs magnétiques générés. Dans les conducteurs plats il y a toujours les défauts de montage, techniques, géométriques et structurels qui apparaissent au processus de leur production et exploitation et qui déterminent leur compatibilité électromagnétique. La concentration de l'énergie électromagnétique autour de ces défauts peut amener l'équipement électrique à bloc. C'est pourquoi l'observation et la mesure des champs magnétiques autour des défauts qui se localisent dans de petits volumes est un problème actuel. Citons quelques études de l'observation de ces champs autour des défauts sur les conducteurs plats et présentons le mécanisme du calcul à l'aide de moire électronique optique.

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