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DETERMINATION OF DISTRIBUTION'S PARAMETERS OF MATERIAL PORTION ON THE FLAT TRAY AT THE INITIAL TIME POINT BY IMPLEMENTATION OF TWO-STAGE FEEDING TECHNOLOGY

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Abstract: The results of analytical definition of parameters distribution of a separate bulk solids portion on the rectangular flat tray by the implementation of two-stage feeding technology are presented in the article. The numerical verification of derived dependences has been performed.

Two-stage feeding technology has been proposed before [1]. The principle of this technology is in transformation of separate portion into continuous flow, this process could be realized with the aid of vibration, in particular [2].

During vibration of bulk particles undergo complicated movement, being moved in both horizontal and vertical axis. The process of transformation of separate portion in continuous flow includes the change in shape of open surface of a separate portion, the movement of the portion gravity center along the tray and the combination of separate portions between each other.

The mathematical model of the process of a separate portion's transformation in a continuous flow [3] has been offered. To use the given model it is necessary to know the initial distribution parameters of a separate portion in a flat tray

It has been proved experimentally that the curve which restricts the portion of bulk from the top represents a half-wave of sinusoid in the longitudinal section (Fig. 1). Upon the movement through the tray, only the shape of the longitudinal section is changing, while its surface area remains constant. Therefore, if we know the portion's surface area in zero moment of time, we could model the section shape's behavior for some period of time.

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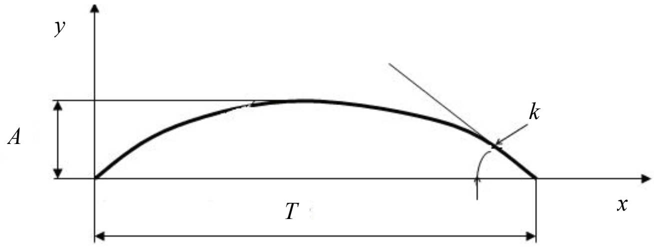

Fig. 1. Longitudinal section of a portion of bulk:
A – amplitude of sinusoid (height of material's portion); *T* – half-period of sinusoid (length of longitudinal section of material's portion); *k* – tangent angle of slope to sinusoid and slope's angle of material

To determine the section area of the portion, we need to know sinusoid parameters: the amplitude and half-period. These values used to be obtained by direct measurements for every experiment [3]. However, if the volume of the portion being loaded and the slope of this type of bulk, one could determine the portions parameters at the zero time point analytically.

Let's consider a separate portion of material on the flat and derive the dependence of amplitude and half-period of sinusoid from the volume of portion and material slope (considering rectangular-shaped flat tray).

Sinusoid equation

$$y(x) = A\cos\left(\frac{\pi x}{2T}\right), \tag{1}$$

where A is amplitude, T is half-period.

Write down expression for tangent's slope angle tangent to sinusoid

$$\frac{d}{dx}\left(A\cos\left(\frac{\pi x}{2T}\right)\right) = -\frac{1}{2}A\sin\left(\frac{\pi x}{2T}\right)\frac{\pi}{T}. \tag{2}$$

Then tangent's slope angle tangent with the base ($x = T$) will be as follows

$$-\frac{1}{2}A\sin\left(\frac{\pi T}{2T}\right)\frac{\pi}{T} = -\frac{1}{2}A\frac{\pi}{T}. \tag{3}$$

It is clear that tangent's slope angle tangent with the base will be equal to tangent of given angle of material's slope k , therefore:

$$k = -\frac{1}{2}A\frac{\pi}{T}. \tag{4}$$

Write down formula to derive portion volume using parameters of curve that describes it

$$V = 2\pi\int_0^T\left(A\cos\left(\frac{\pi x}{2T}\right)\right)xdx = 2\pi\left(\frac{2}{\pi}T^2A - \frac{4}{\pi^2}T^2A\right) = \frac{4}{\pi}T^2A(\pi-2). \tag{5}$$

Expression for amplitude A from equations (4)

$$A = -2\mathrm{tg}(k)\frac{T}{\pi}, \tag{6}$$

and from equations (5)

$$A = \frac{1}{4}V\frac{\pi}{T^2(\pi-2)}. \tag{7}$$

Equalize right parts of equations (6) and (7). Express the half-period T

$$-2\mathrm{tg}(k)\frac{T}{\pi}-\frac{1}{4}V\frac{\pi}{T^2(\pi-2)}=0. \qquad (8)$$

The Solution of equation (8) relative to variable T is 3 roots, later one need to use all roots, selecting positive rational solution

$$T=\begin{cases}\frac{1}{2\mathrm{tg}(k)(\pi-2)}\left[-V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}},\\ \frac{-1}{4\mathrm{tg}(k)(\pi-2)}\left[-V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}}-\frac{1}{4}i\frac{\sqrt{3}}{\mathrm{tg}(k)(\pi-2)}\left[-V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}},\\ \frac{-1}{4\mathrm{tg}(k)(\pi-2)}\left[-V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}}+\frac{1}{4}i\frac{\sqrt{3}}{\mathrm{tg}(k)(\pi-2)}\left[-V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}}.\end{cases} \qquad (9)$$

Therefore, the formula for calculating half-period T through given values only is obtained. Let's get the same solution for the amplitude.

Let's express half-period T through amplitude A and slope angle k from equation (4)

$$T=-\frac{1}{2}A\frac{\pi}{\mathrm{tg}(k)}. \qquad (10)$$

Now express half-period T through amplitude A and volume V from equation (5)

$$T=\begin{cases}\frac{2}{(4A\pi-8A)}\left[A(\pi-2)V\pi\right]^{\frac{1}{2}},\\ \frac{-2}{(4A\pi-8A)}\left[A(\pi-2)V\pi\right]^{\frac{1}{2}}.\end{cases} \qquad (11)$$

Now we need to find the solution for amplitude A which is dependent on the known parameters only, i.e. volume V and slope angle k. To do that, insert equation (11) in equation (10). As equation (11) has two conjugate roots, insertion of any of those will give the same result, so from that, selecting first root:

$$-\frac{1}{2}A\frac{\pi}{tg(k)}-\frac{2}{(4A\pi-8A)}\left[A(\pi-2)V\pi\right]^{\frac{1}{2}}=0; \qquad (12)$$

$$A=\begin{cases}\frac{1}{\pi(\pi-2)}\left[V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}},\\ \frac{-1}{2\pi(\pi-2)}\left[V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}}+\frac{1}{2}i\frac{\sqrt{3}}{\pi(\pi-2)}\left[V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}},\\ \frac{-1}{2\pi(\pi-2)}\left[V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}}-\frac{1}{2}i\frac{\sqrt{3}}{\pi(\pi-2)}\left[V\pi^2\mathrm{tg}(k)^2(\pi-2)^2\right]^{\frac{1}{3}}.\end{cases} \qquad (13)$$

Therefore, formulas for the calculation of amplitude and half-period of the bulk portion from the volume of the portion and the slope angle are obtained. The negative value of the slope angle should be used (because the calculations are based on the right half of the portion) and it should be expressed in radians.

Let's make numerical check; all values will be in reference units. Let volume $V = 10$ and slope angle $k = \mathrm{arctg}(-0{,}3\pi) = 0{,}756$.

Find the solution for half-period T from equation (9) and select positive rational root:

$$T_1 = \frac{1}{2\mathrm{tg}(k)(\pi - 2)}\left[-V\pi^2 \mathrm{tg}(k)^2 (\pi - 2)^2\right]^{\frac{1}{3}} = -1{,}127 - 1{,}953i\,; \tag{14}$$

$$T_2 = \frac{-1}{4\mathrm{tg}(k)(\pi - 2)}\left[-V\pi^2 \mathrm{tg}(k)^2 (\pi - 2)^2\right]^{\frac{1}{3}} -$$

$$-\frac{1}{4}i\frac{\sqrt{3}}{\mathrm{tg}(k)(\pi - 2)}\left[-V\pi^2 \mathrm{tg}(k)^2 (\pi - 2)^2\right]^{\frac{1}{3}} = -1{,}127 + 1{,}953i\,; \tag{15}$$

$$T_3 = \frac{-1}{4\mathrm{tg}(k)(\pi - 2)}\left[-V\pi^2 \mathrm{tg}(k)^2 (\pi - 2)^2\right]^{\frac{1}{3}} +$$

$$+\frac{1}{4}i\frac{\sqrt{3}}{\mathrm{tg}(k)(\pi - 2)}\left[-V\pi^2 \mathrm{tg}(k)^2 (\pi - 2)^2\right]^{\frac{1}{3}} = 2{,}255\,. \tag{16}$$

It is clear that the solution will be $T = 2{,}255$.

Now from equation (7) we will find amplitude A

$$A = \frac{1}{4}V\frac{\pi}{T^2(\pi - 2)} = 1{,}353\,.$$

Let's calculate amplitude A with the equation (13), using only known values:

$$A_1 = \frac{1}{\pi(\pi - 2)}\left[V\pi^2 \mathrm{tg}(k)^2 (\pi - 2)^2\right]^{\frac{1}{3}} = 1{,}353\,;$$

$$A_2 = \frac{-1}{2\pi(\pi - 2)}\left[V\pi^2 \mathrm{tg}(k)^2 (\pi - 2)^2\right]^{\frac{1}{3}} +$$

$$+\frac{1}{2}i\frac{\sqrt{3}}{\pi(\pi - 2)}\left[V\pi^2 \mathrm{tg}(k)^2 (\pi - 2)^2\right]^{\frac{1}{3}} = -0{,}676 + 1{,}172i\,;$$

$$A_3 = \frac{-1}{2\pi(\pi - 2)}\left[V\pi^2 k^2 (\pi - 2)^2\right]^{\frac{1}{3}} - \frac{1}{2}i\frac{\sqrt{3}}{\pi(\pi - 2)}\left[V\pi^2 k^2 (\pi - 2)^2\right]^{\frac{1}{3}} = -0{,}676 - 1{,}172i\,.$$

Positive rational root is $A = 1{,}353$, it fits the previous solution, therefore our calculations are correct.

Now perform the check, taking into account the obtained amplitude and half-period, let's calculate the starting volume from equation (5)

$$V = \frac{4}{\pi} T^2 A(\pi - 2) = \frac{4}{\pi} 2{,}255^2 1{,}353(\pi - 2) = 10 .$$

The calculated values of the volumes are the same, so the dependence derived earlier can be used for the calculation of the distribution parameters of a separate portion in a squared flat tray.

References

1. Пат. 2138783 Российская Федерация, МПК G 01 F 11/00. Способ непрерывного дозирования сыпучих материалов / В.Ф. Першин, С.В. Барышникова ; заявитель и патентообладатель Тамб. гос. техн. ун-т. – № 98110695/28 ; заявл. 04.06.98 ; опубл. 27.09.99, Бюл. № 27. – 4 с.
2. Пат. 2251083 Российская Федерация, МПК G 01 F 11/00. Способ непрерывного дозирования сыпучих материалов и устройство для его осуществления / В.Ф. Першин, С.В. Барышникова, Д.К. Каляпин, А.А. Осипов ; заявитель и патентообладатель Тамб. гос. техн. ун-т. – № 2003109774/28 ; заявл. 07.04.03 ; опубл. 27.04.05, Бюл. № 12. – 11 с.
3. Осипов, А.А. Математическое описание движения сыпучего материала на наклонном вибрирующем лотке / А.А. Осипов, С.В. Першина // Тр. Тамб. гос. техн. ун-та : сб. науч. ст. молодых ученых и студентов. – Тамбов, 2004. – Вып. 15. – С. 45–49.
4. Бронштейн, И.Н. Справочник по математике для инженеров и учащихся втузов / И.Н. Бронштейн, К.А. Семендяев. – 13-е изд., испр. – М. : Наука, 1986. – 544 с.

Определение параметров распределения порции материала на плоском лотке в начальный момент времени при реализации двухстадийной технологии дозирования

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Ключевые слова и фразы: сыпучие материалы; технология двухстадийного дозирования.

Аннотация: Представлены результаты аналитического определения параметров распределения отдельной порции сыпучего материала в прямоугольном плоском лотке при реализации двухстадийной технологии дозирования. Проведена численная проверка полученных зависимостей.
