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LAMINATED COMPOSITE SHELL IN CYLINDRICAL BENDING

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Abstract: This paper presents an efficient method of solving the plane strain problem of elasticity for laminated composite cylindrical shells. The method is based on the new concept of sampling surfaces (SaS) proposed recently by the authors. According to this concept, we introduce inside the *n*th layer I_n not equally spaced SaS parallel to the middle surface of the shell and choose displacements of these surfaces as shell unknowns. Such choice of displacements allows the derivation of strain-displacement relationships, which exactly represent all rigid-body shell motions in any curvilinear surface coordinate system. The latter gives in turn the opportunity to find the solutions of plane strain elasticity for thick and thin laminated cylindrical shells with a prescribed accuracy by using a sufficiently large number of SaS, which are located at layer interfaces and Chebyshev polynomial nodes.

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1. Introduction

A conventional way for developing the higher order shell formulation consists in the expansion of displacements into power series with respect to the transverse coordinate, which referred to the direction normal to the middle surface of the shell. For the approximate representation of the displacement field, it is possible to use finite segments of power series because the principal purpose of the shell theory consists in the derivation of approximate solutions of elasticity. Such a way has been extensively utilized for development of the higher order layer-wise shell models accounting for thickness stretching [1–3]. However, the implementation of this approach for thick laminated shells is not easy to realize, since it is necessary to retain a sufficiently large number of terms in corresponding expansions to obtain the comprehensive results.

More efficient approach for the analysis of laminated shells can be achieved through the use of a SaS method. This method has been recently proposed by the authors for homogeneous shells [4, 5] and laminated plates [6]. In the present paper, we extend the SaS method to laminated composite shells to solve the plane strain problem of elasticity. As SaS denoted by $\Omega^{(n)1}, \Omega^{(n)2},..., \Omega^{(n)I_n}$, we choose outer surfaces and any inner surfaces inside the *n*th layer and introduce displacement vectors $\mathbf{u}^{(n)1}, \mathbf{u}^{(n)2},..., \mathbf{u}^{(n)I_n}$ of these surfaces as fundamental shell unknowns, where I_n is the total number of SaS selected for each layer ($I_n \geq 3$). The index n identifies the belonging of any quantity to the *n*th layer and runs from 1 to *N*, where *N* is the number of layers. Such choice of displacements with the consequent use of Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer allows the derivation of strain-displacement equations, which exactly represent all rigid-body motions of the shell in any convected curvilinear coordinate system.

Unfortunately, the above polynomial interpolation in the thickness direction implemented for equally spaced SaS [4–6] does not work properly with Lagrange polynomials of high degree because of Runge's phenomenon [7]. However, the use of Chebyshev polynomial nodes [8] can help to improve significantly the behaviour of Lagrange polynomials of high degree for which the error will go to zero as $I_n \to \infty$. This fact gives an opportunity to derive the elasticity solutions for thick laminated shells with a prescribed accuracy employing a sufficient number of SaS located at layer interfaces and Chebyshev polynomial nodes.

Consider a thick laminated shell of the thickness *h*. Let the middle surface Ω be described by orthogonal curvilinear coordinates θ_1 and θ_2, which are referred to the lines of principal curvatures of its surface. The coordinate θ_3 is oriented along the unit vector $\boldsymbol{a}_3 = \mathbf{e}_3$ normal to the middle surface. Introduce the following notations: $\mathbf{r} = \mathbf{r}(\theta_1, \theta_2)$ is the position vector of any point of the middle surface; $\boldsymbol{a}_\alpha$ are the base vectors of the middle surface given by

$$\boldsymbol{a}_\alpha = \mathbf{r}_{,\alpha} = A_\alpha \mathbf{e}_\alpha , \qquad (1)$$

where $\mathbf{e}_\alpha$ are the orthonormal base vectors; A_α are the coefficients of the first fundamental form; $\theta_3^{(n)i_n}$ are the transverse coordinates of SaS of the *n*th layer expressed as

$$\theta_3^{(n)1} = \theta_3^{[n-1]}, \quad \theta_3^{(n)I_n} = \theta_3^{[n]},$$

$$\theta_3^{(n)m_n} = \frac{1}{2}(\theta_3^{[n-1]} + \theta_3^{[n]}) - \frac{1}{2} h_n \cos\left(\pi \frac{2m_n - 3}{2(I_n - 2)}\right), \qquad (2)$$

where $\theta_3^{[n-1]}$ and $\theta_3^{[n]}$ are the transverse coordinates of the bottom and top surfaces $\Omega^{[n-1]}$ and $\Omega^{[n]}$ of the *n*th layer (Fig. 1) such that $\theta_3^{[0]} = -h/2$ and $\theta_3^{[N]} = h/2$; $h_n = \theta_3^{[n]} - \theta_3^{[n-1]}$ is the thickness of the *n*th layer; $\mathbf{R} = \mathbf{r} + \theta_3 \mathbf{e}_3$ is the position vector of any point in the shell body; $\mathbf{R}^{(n)i_n} = \mathbf{r} + \theta_3^{(n)i_n} \mathbf{e}_3$ are the position vectors of SaS of the *n*th layer; $\mathbf{g}_i$ are the base vectors in the shell body defined as

$$\mathbf{g}_\alpha = \mathbf{R}_{,\alpha} = A_\alpha c_\alpha \mathbf{e}_\alpha, \quad \mathbf{g}_3 = \mathbf{R}_{,3} = \mathbf{e}_3 , \qquad (3)$$

where $c_\alpha = 1 + k_\alpha \theta_3$ are the components of the shifter tensor; k_α are the principal curvatures of the middle surface; $\mathbf{g}_i^{(n)i_n}$ are the base vectors of SaS of the *n*th layer given by

$$\mathbf{g}_\alpha^{(n)i_n} = \mathbf{R}_{,\alpha}^{(n)i_n} = A_\alpha c_\alpha^{(n)i_n} \mathbf{e}_\alpha, \quad \mathbf{g}_3^{(n)i_n} = \mathbf{e}_3 , \qquad (4)$$

where $c_\alpha^{(n)i_n} = 1 + k_\alpha \theta_3^{(n)i_n}$ are the components of the shifter tensor at SaS. Here and in the following developments, $(\ldots)_{,i}$ stands for the partial derivatives with respect to coordinates θ_i; the index m_n identifies the belonging of any quantity to the inner SaS of the *n*th layer and runs from 2 to $I_n - 1$, whereas the indices i_n, j_n, k_n describe all SaS of the *n*th layer and run from 1 to I_n; Greek tensorial indices α, β range from 1 to 2; Latin tensorial indices i, j, k, l range from 1 to 3.

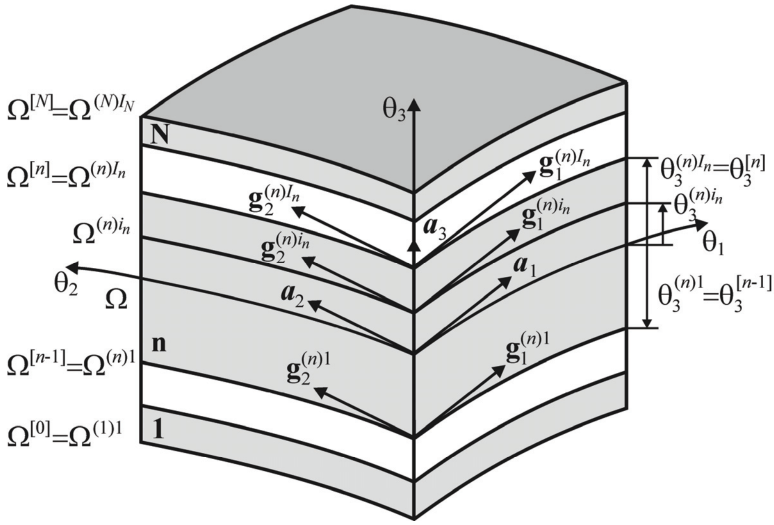


Fig. 1. Geometry of the thick laminated shell

It should be mentioned that transverse coordinates of inner SaS (2) coincide with the nodes of Chebyshev polynomials [7]. This fact has a great meaning for a convergence of the SaS method.

2. Three-dimensional kinematic description of laminated shell

A position vector of the deformed shell is written as

$$\overline{\mathbf{R}} = \mathbf{R} + \mathbf{u}, \tag{5}$$

where $\mathbf{u}$ is the displacement vector, which is always measured in accordance with the total Lagrangian formulation from the initial configuration to the current configuration directly. In particular, the position vectors of SaS of the nth layer are

$$\overline{\mathbf{R}}^{(n)i_n} = \mathbf{R}^{(n)i_n} + \mathbf{u}^{(n)i_n}, \tag{6}$$

$$\mathbf{u}^{(n)i_n} = \mathbf{u}(\theta_3^{(n)i_n}), \tag{7}$$

where $\mathbf{u}^{(n)i_n}(\theta_1, \theta_2)$ are the displacement vectors of SaS of the nth layer (Fig. 2). Due to continuity conditions, we have

$$\mathbf{u}^{(1)1} = \mathbf{u}^{[0]}, \quad \mathbf{u}^{(N)I_N} = \mathbf{u}^{[N]},$$

$$\mathbf{u}^{(m)I_m} = \mathbf{u}^{(m+1)1} = \mathbf{u}^{[m]}, \tag{8}$$

where $\mathbf{u}^{[m]}(\theta_1, \theta_2)$ are the displacement vectors of layer interfaces $\Omega^{[m]}$ $(m = 1, 2, ..., N-1)$.

The base vectors in the current shell configuration are defined as

$$\overline{\mathbf{g}}_i = \overline{\mathbf{R}}_{,i} = \mathbf{g}_i + \mathbf{u}_{,i}. \tag{9}$$

In particular, the base vectors of deformed SaS of the nth layer are

$$\overline{\mathbf{g}}_\alpha^{(n)i_n} = \overline{\mathbf{R}}_{,\alpha}^{(n)i_n} = \mathbf{g}_\alpha^{(n)i_n} + \mathbf{u}_{,\alpha}^{(n)i_n}, \quad \overline{\mathbf{g}}_3^{(n)i_n} = \overline{\mathbf{g}}_3(\theta_3^{(n)i_n}) = \mathbf{e}_3 + \boldsymbol{\beta}^{(n)i_n}, \tag{10}$$

$$\boldsymbol{\beta}^{(n)i_n} = \mathbf{u}_{,3}(\theta_3^{(n)i_n}), \tag{11}$$

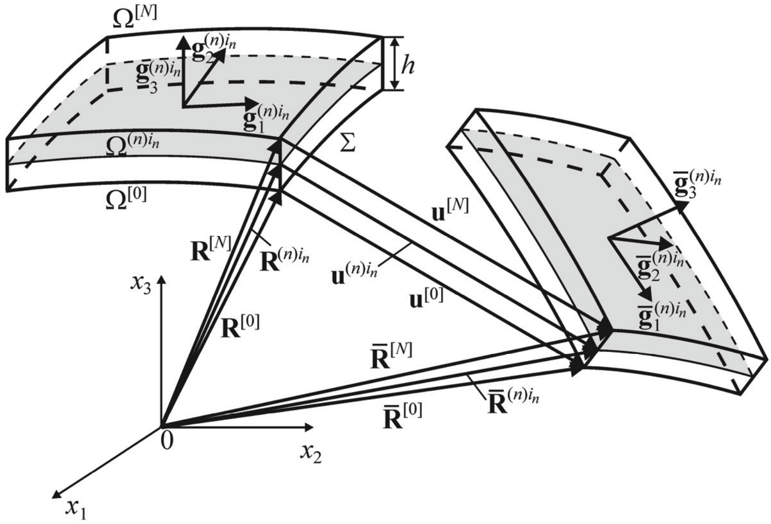


Fig. 2. Initial and current configurations of the shell

where $\boldsymbol{\beta}^{(n)i_n}(\theta_1, \theta_2)$ are the values of the derivative of the 3D displacement vector with respect to coordinate θ_3 at SaS.

The Green–Lagrange strain tensor in an orthogonal curvilinear coordinate system [4] can be written as

$$2\varepsilon_{ij} = \frac{1}{A_i A_j c_i c_j}\left(\bar{\mathbf{g}}_i \cdot \bar{\mathbf{g}}_j - \mathbf{g}_i \cdot \mathbf{g}_j\right), \tag{12}$$

where $A_3 = 1$ and $c_3 = 1$. In particular, the Green-Lagrange strains at SaS are

$$2\varepsilon_{ij}^{(n)i_n} = 2\varepsilon_{ij}(\theta_3^{(n)i_n}) = \frac{1}{A_i A_j c_i^{(n)i_n} c_j^{(n)i_n}}\left(\bar{\mathbf{g}}_i^{(n)i_n} \cdot \bar{\mathbf{g}}_j^{(n)i_n} - \mathbf{g}_i^{(n)i_n} \cdot \mathbf{g}_j^{(n)i_n}\right). \tag{13}$$

Substituting (4) and (10) into the strain-displacement relationships (13) and discarding the non-linear terms, one derives

$$2\varepsilon_{\alpha\beta}^{(n)i_n} = \frac{1}{A_\alpha c_\alpha^{(n)i_n}}\mathbf{u}_{,\alpha}^{(n)i_n} \cdot \mathbf{e}_\beta + \frac{1}{A_\beta c_\beta^{(n)i_n}}\mathbf{u}_{,\beta}^{(n)i_n} \cdot \mathbf{e}_\alpha,$$

$$2\varepsilon_{\alpha 3}^{(n)i_n} = \boldsymbol{\beta}^{(n)i_n} \cdot \mathbf{e}_\alpha + \frac{1}{A_\alpha c_\alpha^{(n)i_n}}\mathbf{u}_{,\alpha}^{(n)i_n} \cdot \mathbf{e}_3, \quad \varepsilon_{33}^{(n)i_n} = \boldsymbol{\beta}^{(n)i_n} \cdot \mathbf{e}_3. \tag{14}$$

Next, we represent the displacement vectors $\mathbf{u}^{(n)i_n}$ and $\boldsymbol{\beta}^{(n)i_n}$ in the reference surface frame $\mathbf{e}_i$ as follows:

$$\mathbf{u}^{(n)i_n} = \sum_i u_i^{(n)i_n}\mathbf{e}_i, \tag{15}$$

$$\boldsymbol{\beta}^{(n)i_n} = \sum_i \beta_i^{(n)i_n}\mathbf{e}_i. \tag{16}$$

Using (15) and well-known formulas [9] for derivatives of unit vectors $\mathbf{e}_i$ with respect to orthogonal curvilinear coordinates θ_α, we have

$$\frac{1}{A_\alpha}\mathbf{u}^{(n)i_n}_{,\alpha} = \sum_i \lambda^{(n)i_n}_{i\alpha}\mathbf{e}_i, \tag{17}$$

where

$$\lambda^{(n)i_n}_{\alpha\alpha} = \frac{1}{A_\alpha}u^{(n)i_n}_{\alpha,\alpha} + B_\alpha u^{(n)i_n}_\beta + k_\alpha u^{(n)i_n}_3 \quad \text{for} \quad \beta \neq \alpha,$$

$$\lambda^{(n)i_n}_{\beta\alpha} = \frac{1}{A_\alpha}u^{(n)i_n}_{\beta,\alpha} - B_\alpha u^{(n)i_n}_\alpha \quad \text{for} \quad \beta \neq \alpha,$$

$$\lambda^{(n)i_n}_{3\alpha} = \frac{1}{A_\alpha}u^{(n)i_n}_{3,\alpha} - k_\alpha u^{(n)i_n}_\alpha. \tag{18}$$

Inserting (16) and (17) into the strain-displacement relationships (14), we arrive at the following equations:

$$2\varepsilon^{(n)i_n}_{\alpha\beta} = \frac{1}{c^{(n)i_n}_\beta}\lambda^{(n)i_n}_{\alpha\beta} + \frac{1}{c^{(n)i_n}_\alpha}\lambda^{(n)i_n}_{\beta\alpha},$$

$$2\varepsilon^{(n)i_n}_{\alpha 3} = \beta^{(n)i_n}_\alpha + \frac{1}{c^{(n)i_n}_\alpha}\lambda^{(n)i_n}_{3\alpha}, \qquad \varepsilon^{(n)i_n}_{33} = \beta^{(n)i_n}_3. \tag{19}$$

3. Displacement and strain distributions in thickness direction

Up to this moment, no assumptions concerning displacement and strain fields have been made. We start now with the first fundamental assumption of the proposed higher order layer-wise shell theory. Let us assume that the displacements are distributed through the thickness of the *n*th layer as follows:

$$u^{(n)}_i = \sum_{i_n} L^{(n)i_n} u^{(n)i_n}_i, \qquad \theta^{[n-1]}_3 \le \theta_3 \le \theta^{[n]}_3, \tag{20}$$

where $L^{(n)i_n}(\theta_3)$ are the Lagrange polynomials of degree $I_n - 1$ expressed as

$$L^{(n)i_n} = \prod_{j_n \neq i_n} \frac{\theta_3 - \theta^{(n)j_n}_3}{\theta^{(n)i_n}_3 - \theta^{(n)j_n}_3}. \tag{21}$$

The use of relations (11), (16) and (20) yields

$$\beta^{(n)i_n}_i = \sum_{j_n} M^{(n)j_n}(\theta^{(n)i_n}_3)u^{(n)j_n}_i, \tag{22}$$

where $M^{(n)j_n} = L^{(n)j_n}_{,3}$ are the derivatives of Lagrange polynomials. The values of these derivatives at SaS are calculated as

$$M^{(n)j_n}\left(\theta^{(n)i_n}_3\right) = \frac{1}{\theta^{(n)j_n}_3 - \theta^{(n)i_n}_3} \prod_{k_n \neq i_n, j_n} \frac{\theta^{(n)i_n}_3 - \theta^{(n)k_n}_3}{\theta^{(n)j_n}_3 - \theta^{(n)k_n}_3} \quad \text{for} \quad j_n \neq i_n,$$

$$M^{(n)i_n}\left(\theta^{(n)i_n}_3\right) = -\sum_{j_n \neq i_n} M^{(n)j_n}\left(\theta^{(n)i_n}_3\right). \tag{23}$$

Thus, the key functions $\beta^{(n)i_n}_i$ of the proposed layer-wise shell theory are represented according to (22) as a linear combination of displacements of SaS of the *n*th layer $u^{(n)j_n}_i$.

The following step consists in a choice of the correct approximation of strains through the thickness of the nth layer. It is apparent that the optimal solution of the problem is to choose the strain distribution, which is similar to the displacement distribution (20), that is,

$$\varepsilon_{ij}^{(n)} = \sum_{i_n} L^{(n)i_n} \varepsilon_{ij}^{(n)i_n}, \qquad \theta_3^{[n-1]} \le \theta_3 \le \theta_3^{[n]}. \tag{24}$$

Strain-displacement relationships (14) and (24) are invariant under rigid-body motions of a laminated shell in any curvilinear surface coordinate system. The idea of a proof can be found in [10, 11]. The ability of proposed strain-displacement relationships exactly represent all rigid-body shell motions admits the development of exact geometry solid-shell elements [12, 13]. The term "exact geometry" reflects the fact that the parametrization of the reference surface is known and, therefore, coefficients of the first and second fundamental forms of the surface can be taken exactly at each element node.

4. Total potential energy of laminated shell

Inserting strains (24) in the total potential energy of a laminated shell and introducing stress resultants [4]

$$H_{ij}^{(n)i_n} = \int_{\theta_3^{[n-1]}}^{\theta_3^{[n]}} \sigma_{ij}^{(n)} L^{(n)i_n} c_1 c_2 d\theta_3 , \tag{25}$$

one derives

$$\Pi = \iint_{\Omega} \left[\frac{1}{2} \sum_{n} \sum_{i_n} \sum_{i,j} H_{ij}^{(n)i_n} \varepsilon_{ij}^{(n)i_n} - \sum_{i} \left(c_1^{[N]} c_2^{[N]} p_i^{[N]} u_i^{[N]} - \right.\right.$$

$$\left.\left. - c_1^{[0]} c_2^{[0]} p_i^{[0]} u_i^{[0]} \right) \right] A_1 A_2 d\theta_1 d\theta_2 - W_{\Sigma} , \tag{26}$$

where $c_{\alpha}^{[0]} = 1 + k_{\alpha}\theta_3^{[0]}$ and $c_{\alpha}^{[N]} = 1 + k_{\alpha}\theta_3^{[N]}$ are the components of the shifter tensor at bottom and top surfaces $\Omega^{[0]}$ and $\Omega^{[N]}$; $p_i^{[0]}$ and $p_i^{[N]}$ are the loads acting on bottom and top surfaces of a shell; W_{Σ} is the work done by external loads applied to the boundary surface Σ.

For simplicity, we restrict ourselves to the case of linear elastic materials. The natural choice for constitutive equations is the generalized Hook's law:

$$\sigma_{ij}^{(n)} = \sum_{k,\ell} C_{ijk\ell}^{(n)} \varepsilon_{k\ell}^{(n)}, \qquad \theta_3^{[n-1]} \le \theta_3 \le \theta_3^{[n]}. \tag{27}$$

Substituting stresses (27) in (25) and accounting for the strain distribution (24), we have

$$H_{ij}^{(n)i_n} = \sum_{j_n} \sum_{k,\ell} D_{ijk\ell}^{(n)i_n j_n} \varepsilon_{k\ell}^{(n)j_n} , \tag{28}$$

where

$$D_{ijk\ell}^{(n)i_n j_n} = C_{ijk\ell}^{(n)} \int_{\theta_3^{[n-1]}}^{\theta_3^{[n]}} L^{(n)i_n} L^{(n)j_n} c_1 c_2 d\theta_3 . \tag{29}$$

5. Exact solution for laminated cylindrical shell

Let us consider a cylindrical bending of a simply supported laminated cylindrical shell of the radius R subjected to the sinusoidally distributed load acting on the top surface

$$p_3^{[N]} = p_0 \sin m\theta_2, \quad \theta_2 \in [0, \theta_*], \tag{30}$$

where θ_2 is the circumferential coordinate of the middle surface.

The analytical solution of the problem satisfying the boundary conditions can be written as

$$u_1^{(n)i_n} = 0, \quad u_2^{(n)i_n} = u_{20}^{(n)i_n} \cos m\theta_2, \quad u_3^{(n)i_n} = u_{30}^{(n)i_n} \sin m\theta_2. \tag{31}$$

Substituting (30) and (31) in the total potential energy (26) with $W_\Sigma = 0$ and allowing for relations (18), (19) and (28), one finds

$$\Pi = \Pi\left(u_{20}^{(n)i_n}, u_{30}^{(n)i_n}\right). \tag{32}$$

Invoking the principle of the minimum total potential energy, we arrive at the system of linear algebraic equations

$$\frac{\partial \Pi}{\partial u_{20}^{(n)i_n}} = 0, \quad \frac{\partial \Pi}{\partial u_{30}^{(n)i_n}} = 0 \tag{33}$$

of order $2\left(\sum_n I_n - N + 1\right)$. The linear system (33) can be solved by using a method of Gaussian elimination.

The described algorithm was performed with the Symbolic Math Toolbox, which incorporates symbolic computations into the numeric environment of MATLAB. This gave the possibility to derive the exact solutions of plane strain elasticity for laminated composite cylindrical shells with a prescribed accuracy.

The geometrical and mechanical parameters of a three-ply cylindrical shell are taken to be $R = 10$, $m = 3$, $\theta_* = \pi/3$ and $E_L = 25E_T$, $G_{LT} = 0{,}5E_T$, $G_{TT} = 0{,}2E_T$, $E_T = 10^6$, $\nu_{LT} = \nu_{TT} = 0{,}25$, where subscripts L and T refer to the fiber and transverse directions of the ply, and L-direction coincides with θ_1-direction. Here, we study a shell with the lamination scheme $h_n = h/3$ and $[90^\circ/0^\circ/90^\circ]$. To compare the results derived with Ren's exact solution of the plane strain elasticity [14], the following dimensionless variables are introduced:

$$U_3 = 100E_T h^3 u_3(\pi/6, z)/p_0 R^4, \quad S_{11} = 100h^2 \sigma_{11}(\pi/6, z)/p_0 R^2,$$

$$S_{22} = h^2 \sigma_{22}(\pi/6, z)/p_0 R^2, \quad S_{23} = 10h\sigma_{23}(0, z)/p_0 R,$$

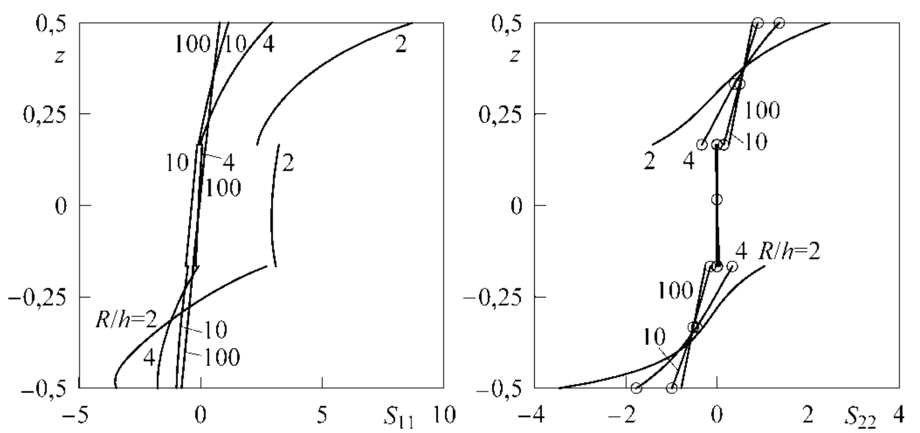
$$S_{33} = \sigma_{33}(\pi/6, z)/p_0, \quad z = \theta_3/h. \tag{34}$$

The data listed in Tables show that the SaS technique allows the derivation of exact solutions for thick and thin shells with a prescribed accuracy using a sufficient number of SaS. Figs. 3 present the distribution of stresses through the thickness of the shell for different values of the slenderness ratio R/h by choosing 9 SaS for each layer. These results demonstrate convincingly the high potential of the proposed layer-wise shell formulation. This is due to the fact that boundary conditions on the bottom and top surfaces and continuity conditions at layer interfaces for transverse stresses are satisfied precisely without integration of the equilibrium equations of elasticity, i.e. only constitutive equations (27) are applied. It is important that the enhanced SaS method provides the uniform convergence that is impossible with equally spaced SaS [4–6].

Table 1

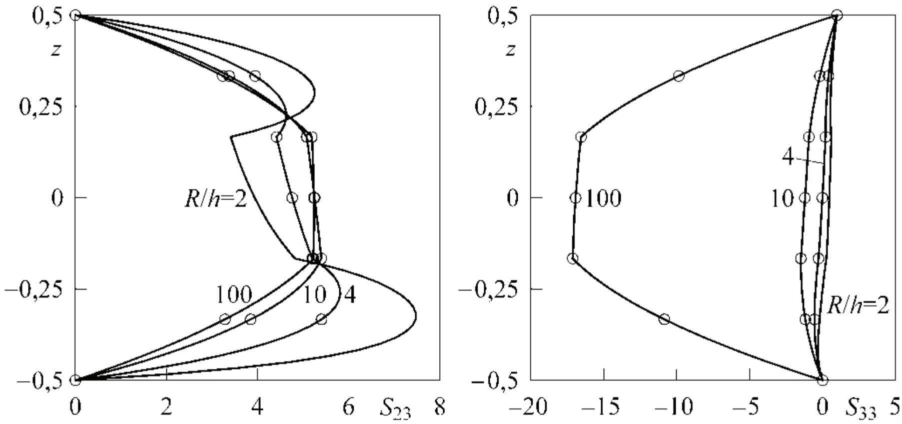
Results for a thick three-ply cylindrical shell

I_n	$U_3(0)$	$S_{11}(-0,5)$	$S_{11}(0,5)$	$S_{22}(-0,5)$	$S_{22}(0,5)$	$S_{23}(0)$	$S_{33}(0,5)$
			R/h = 2				
3	13,853	−5,2469	7,7902	−2,9025	2,3088	3,7576	0,8770
5	14,360	−3,8054	8,6232	−3,4761	2,4618	3,353	0,9858
7	14,357	−3,4951	8,7140	−3,4672	2,4631	3,9352	1,0001
9	14,357	−3,4709	8,7140	−3,4670	2,4631	3,9352	1,0001
11	14,357	−3,4678	8,7133	−3,4669	2,4631	3,9352	1,0000
13	14,357	−3,4672	8,7132	−3,4669	2,4631	3,9352	1,0000
Ren [14]	14,36	−3,47	8,71	−3,467	2,463	3,94	1,0000
			R/h = 4				
3	4,5060	−2,3348	2,7340	−1,7382	1,3483	4,7085	0,8868
5	4,5816	−1,7830	2,9208	−1,7717	1,3670	4,7574	0,9945
7	4,5815	−1,7730	2,9301	−1,7714	1,3670	4,7573	1,0004
9	4,5815	−1,7717	2,9296	− 1,7714	1,3670	4,7573	1,0001
11	4,5815	−1,7715	2,9296	−1,7714	1,3670	4,7573	1,0000
Ren [14]	4,57	−1,77	2,93	−1,772	1,367	4,76	1,0000
			R/h = 100				
3	0,78578	−0,79062	0,77808	−0,78661	0,77907	5,2258	0,3973
5	0,78578	−0,78656	0,78162	−0,78656	0,77912	5,2341	0,9986
7	0,78578	−0,78656	0,78162	−0,78656	0,77912	5,2341	1,0002
9	0,78578	−0,78656	0,78162	−0,78656	0,77912	5,2341	1,0000
Ren [14]	0,787	−0,79	0,78	−0,786	0,781	5,23	1,0000



a)

Fig. 3. Distribution of stresses through the thickness of the three-ply shell:
—— – present analysis; ○ – Ren's solution [14];
a – S_{11}, S_{22}



b)

Fig. 3. (continue):
$b - S_{23}, S_{33}$

6. Conclusion

An efficient method of solving the plane strain problem of elasticity for laminated composite shells has been proposed. It is based on the new technique of SaS located at the Chebyshev polynomial nodes inside the shell body and layer interfaces as well. The stress analysis of composite shells is based on the complete constitutive equations and gives an opportunity to obtain the exact solutions of plane strain elasticity for thick and thin laminated cylindrical shells with a prescribed accuracy.

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Цилиндрический изгиб слоистой композитной оболочки

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Ключевые слова и фразы: метод выборочных поверхностей; перекрестно армированный композит; плоская деформация; цилиндрическая оболочка.

Аннотация: Статья представляет эффективный метод решения плоской задачи теории упругости для слоистых композитных цилиндрических оболочек. Метод основан на новой концепции выборочных поверхностей SaS, предложенной авторами в предыдущих работах. Согласно этой концепции внутри n-го слоя вводятся I_n произвольным образом расположенных выборочных поверхностей, параллельных срединной поверхности оболочки. В качестве искомых функций выбираются перемещения этих поверхностей. Такой выбор перемещений дает возможность получать деформационные соотношения, которые представляют точно движение оболочки как жесткого тела в системе криволинейных поверхностных координат. Это в свою очередь дает возможность находить решение плоской задачи теории упругости для толстых и тонких слоистых цилиндрических оболочек с заданной точностью, используя достаточно большое число SaS, которые размещаются на поверхностях раздела слоев и в узловых точках полинома Чебышёва.

Zylindrische Biegung der geschichteten Kompositumhüllung

Zusammenfassung: Im Artikel ist die wirksame Methode der Lösung der flachen Aufgabe der Theorie der Elastizität für die geschichteten zylindrischen Kompositumhüllungen vorgelegt. Die Methode ist auf der neuen von den Autoren vor kurzem vorgeschlagenen Konzeption der stichprobenartigen Oberflächen (SaS) gegründet. Laut dieser Konzeption werden innerhalb der *n*-Schicht die in einer willkürlichen Weise angeordneten SaS parallel der Mitteloberfläche der Hülle eingeführt und als gesuchte Funktionen werden die Umstellungen dieser Oberflächen gewählt. Solche Auswahl der Umstellungen ermöglicht es, die Deformationsverhältnisse zu bekommen, die genau die Bewegung der Hülle wie des harten Körpers im System der krummlinigen oberflächlichen Koordinaten vorlegen. Es ermöglicht seinerseits, die Lösung der flachen Aufgabe der Theorie der Elastizität für die dicken und feinen geschichteten zylindrischen Hüllen mit der aufgegebenen Genauigkeit zu finden, verwendend die genug große Zahl von SaS, die auf den Oberflächen der Abteilung der Schichten und in den Knotenpunkten des Polynoms von Tchebyschew aufgestellt werden.

Courbure cylindrique de l'enveloppe feulletée composite

Résumé: L'article présente une méthode efficace de la solution du problème plane de la théorie de l'élastisité pour les enveloppes cylindriques feuiletées composites. La méthode est fondée sur une nouvelle conception des surfaces sélectives (SaS), proposée par les auteurs. Selon cette conception à l'intérieur de la couche *n* sont introduites les SaS I_n situées librement parallèles à la surface médianne de l'enveloppe et en qualité des fonctions recherchées sont choisis les déplacements de ces surfaces. Un tel choix de déplacements donne la possibilité de recevoir les relations de déformation qui présente de la manière précise le mouvement de l'enveloppe comme corps solide dans le système des coordonnées superficielles curvilignes.

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