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DEVELOPMENT OF INTERVAL MATHEMATICAL MODELS FOR HEAT TRANSFER PROCESSES

T.A. Frolova, S.V. Frolov, D.S. Tulyakov

Tambov State Technical University, Tambov

Represented by Doctor of Technical Sciences, Professor E.N. Tugolukov

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Abstract: The article argues that effective methods of presenting the undetermined parameters of mathematical models are interval numbers. The algorithms for solving the interval mathematical models of statics and dynamics with lumped and distributed parameters are discussed. The algorithms are based on solving optimization problems. The implementation of the algorithms is exemplified by the model of the firing process of the material in a rotary furnace, in which the information about some of the parameters is given in the form of interval numbers. The presented method can effectively solve the problems of designing and technological process control in the conditions of uncertainty.

Introduction

Researchers have accumulated a large experience in developing analytical mathematical models of technological processes [1, 2]. The experience is based on a thorough theoretical analysis of physical and chemical processes occurring in the object being researched. In the derivation of analytical mathematical models the fundamental laws of conservation of matter and energy, as well as the kinetics of the processes of heat-mass transfer, and chemical transformations are used. This allows such models to adequately describe the processes in a wide range of input and control actions.

Alongside with the significant advantages which analytical mathematical models provide, they have some disadvantages associated with the inability to determine the exact values of some of their parameters. Further, let's consider such parameters uncertain and define them as a vector $v = (v_1, v_2, ..., v_i, ..., v_p)$,

Фролова Татьяна Анатольевна – кандидат технических наук, доцент кафедры «Автоматизированное проектирование технологического оборудования», e-mail: frolova@gaps.tstu.ru; Фролов Сергей Владимирович – доктор технических наук, профессор, заведующий кафедрой «Биомедицинская техника»; Туляков Дмитрий Сергеевич – аспирант кафедры «Автоматизированное проектирование технологического оборудования», ТамбГТУ, г. Тамбов.

$v \in R^p$, where R^p – the Euclidean space dimension p. More often, in chemical engineering processes, uncertain parameters include physical and chemical constants, coefficients of heat and mass transfer, heat conduction, the rate of chemical reactions, as well as concentrations of substances in the input flows, etc. Uncertain parameters of mathematical models can also include constructive characteristics of the equipment, for example, the heat exchange surface, the internal volume of the reaction chamber, etc.

There are several approaches to the disclosure of uncertainties. The widely used approach is a probabilistic approach [3], in which uncertain parameters v_i, $i=\overline{1,p}$, are characterized by functions of distribution density $p_i(v_i)$, $i=\overline{1,p}$. Mathematical models, which include such parameters, are called probabilistic. In this case, the distribution functions $p_i(v_i)$, $i=\overline{1,p}$ are constructed on the basis of statistical data about the behavior of the stochastic parameters $v_i\left(i=\overline{1,p}\right)$. However, this approach is difficult to use because of a large number of experiments during the course of the process for determining the parameters of distributions of stochastic values.

Another approach involves the use of fuzzy logic [4] and refers to the sphere of subjective information. Uncertain parameters v_i, $i=\overline{1,p}$, are characterized by membership functions $\mu_i(v_i)$, $i=\overline{1,p}$, which are constructed on the basis of interviews with experts. Models in which uncertain parameters are characterized by membership functions, are called fuzzy mathematical models. The disadvantage of this approach is that reliable construction of membership functions requires the opinion of several experts. This is not always possible.

1. The concept of an interval model

In practice, most often the uncertainty parameter v_i is specified as an interval parameter (interval number)

$$[v_i]=\left[\underline{v_i} \le v_i \le \overline{v_i},\ \underline{v_i} \le \overline{v_i}\right]=\left[\underline{v_i},\overline{v_i}\right]\equiv \mathrm{mid}\,[v_i]\pm\frac{\Delta_i}{2},\quad i=\overline{1,p},$$

where $\underline{v_i}, \overline{v_i}$ are the lower and upper boundaries of parameters v_i; $\mathrm{mid}\left[v_i\right]$ is class mark $[v_i]$ (Fig. 1):

$$\mathrm{mid}\,[v_i]=\left(\underline{v_i}+\overline{v_i}\right)/2; \tag{1}$$

value Δ_i is the interval which is determined by

$$\Delta_i=\overline{v_i}-\underline{v_i}. \tag{2}$$

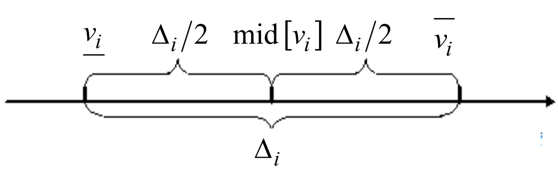


Fig. 1. Image of interval parameter on the number scale

It is assumed that the probability, or any other characteristics, specifying the exact location of the parameter inside or on the boundary of the range Δ_i are not available. Obviously, the interval numbers $[v_i]$ contain minimum of information about the uncertain parameters. Parameters v_i can be either of stochastic or deterministic nature. The uncertainty of the parameters of a deterministic nature can be caused by a lack of knowledge of their exact values.

Mathematical models with such parameters are called interval models. Let's define an interval mathematical model in the form of an operator

$$[y] = \mathrm{M}(u, x, [v]),$$
$$u \in U, \ x \in X, \ y \in Y, \ v \in V, \tag{3}$$

where $[y]$, $[v]$ – interval vectors, defined as: $[y] = ([y_1], \ldots, [y_i], \ldots, [y_m])$, $[v] = ([v_1], \ldots, [v_i], \ldots, [v_p])$; X – space of input values; U – space of admissible control effects; Y – space of output values; V – the space of uncertain parameters.

To solve the problems of design and process control it is required to find an interval vector of the output parameters $[y]$ of the mathematical model (3).

To solve the interval mathematical model $[y] = \mathrm{M}(u, x, [v])$, means for a given vector u, x interval vector $[v] = ([v_1], \ldots, [v_i], \ldots, [v_p])$ to find a vector $[y] = ([y_1], ..., [y_j], ..., [y_m])$ that is defined by the vector of lower and upper boundaries $\underline{y} = (\underline{y_1}, ..., \underline{y_j}, ..., \underline{y_m})$, $\overline{y} = (\overline{y_1}, ..., \overline{y_j}, ..., \overline{y_m})$.

According to the classification [2], mathematical models of processes are divided into static and dynamic models, which in their turn are classified into models with lumped and distributed parameters, i.e. into four classes of mathematical models. Each class has its own algorithm for solving the interval model.

2. The method for solution of interval static model with lumped parameters

An interval static model with lumped parameters is defined by the equations of the form:

$$\forall v \in [v] : \mathrm{M}(y, u, x, v) = 0, \tag{4}$$

where y, u, x, v belong to the Euclidean spaces.

Lower boundaries $\underline{y_j}$, $j = \overline{1, m}$, are defined by to solving optimization problems:

$$\underline{y_j} = \arg \min_{\substack{v_i \in [v_i] \\ \forall i \in F}} y_j, \quad j = \overline{1, m}, \tag{5}$$
$$\forall v \in [v] : \ \mathrm{M}(y, u, x, v) = 0,$$

where F – the set of indices.

Upper boundaries $\overline{y_j}$ are determined by solving the optimization problems:

$$\overline{y_j} = \arg\max_{\substack{v_i \in [v_i] \\ \forall i \in F}} y_j, \quad j = \overline{1,m}, \tag{6}$$

$$\forall v \in [v]: \mathrm{M}(y,u,x,v) = 0.$$

To reduce the number of elements in the set F further investigation of dependence $y_j = y_j(v_i)$, $i = \overline{1,p}$, $j = \overline{1,m}$, determined from the model $[y] = \mathrm{M}(u,x,[v])$ is done.

In the first stage the rule for calculating limits $\underline{y_j}, \overline{y_j}$ is set. To do this, in points $v_{i(k)}$, the corresponding values $y_{kj}^{(i)}$, $k = \overline{1,K_i}$ (Fig. 2, *a*), i.e. are calculated a sequence of

$$\left\{ y_{1j}^{(i)}, y_{2j}^{(i)}, ..., y_{kj}^{(i)}, ..., y_{K_i j}^{(i)} \right\} \tag{7}$$

is generated.

Let's denote the set of indices for which the sequence (7) is monotone, as L_j, $L_j \in F$, $j = \overline{1,m}$.

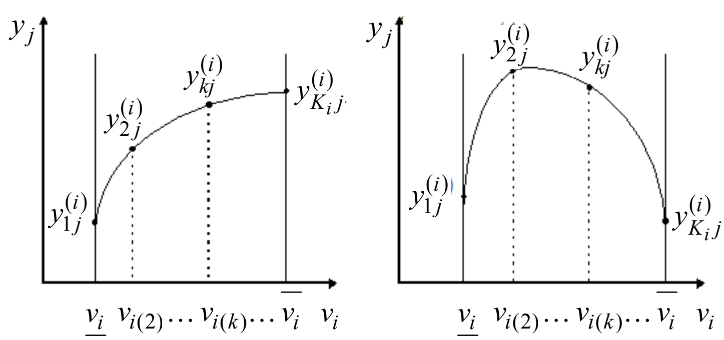
We introduce the notation $\hat{\underline{v}}_i$, $i \in L_j$, in which

$$\underline{y_j}^{(i)} = y_j\left(\hat{\underline{v}}_i\right). \tag{8}$$

This parameter $\hat{\underline{v}}_i$ corresponds to the boundary $\underline{v}_i$, if the sequence (7) is monotonically increasing, and $\hat{\underline{v}}_i$ corresponds to the boundary $\overline{v_i}$, if the sequence (7) is monotonically decreasing. Similarly, the designation $\hat{\overline{v}}_i$, $i \in L_j$, determines the upper boundaries of the dependence

$$\overline{y_j}^{(i)} = y_j\left(\hat{\overline{v}}_i\right). \tag{9}$$

In general, to establish conditions for the validity $i \in L_j$ of the parameter v_i a set of sequences (7) with different random values $v_l \in [v_l]$, $\forall l \in F$, $l \neq i$, $u \in U$, $x \in X$ must be generated. If all the generated sequences (7) for given v_i



a) *b*)

Fig. 2. Monotonic (*a*) and nonmonotonic (*b*) dependence $y_j(v_i)$

are monotonous, only then $i \in L_j$. According to numerous studies, to establish the membership of *i* to the set L_j it is enough to build only one sequence (7), where all the other uncertain parameters in the calculations take the average values $\forall l \in F: v_l = \mathrm{mid}[v_l]$, $l \neq i$ the components of the vectors u, x have any fixed values that satisfy the conditions $u \in U$, $x \in X$. Discretization is chosen interval Δh_i apriori on the basis of preliminary calculations. If $i \notin L_j$, then sequence (7) is nonmonotonic (Fig. 2, *b*).

The second stage of the study of dependence the $y_j^{(i)} = y_j(v_i)$ for the output parameter value y_j the interval Δ_i is to determine.

$$\left| \overline{y_j}^{(i)} - \underline{y_j}^{(i)} \right| > \varepsilon_j^i, \tag{10}$$

where ε_j^i – tolerable error.

Here $\overline{y_j}^{(i)}$, $\underline{y_j}^{(i)}$ are determined from (8), (9), if $i \in L_j$ otherwise, from the decisions of optimization problems $\overline{y_j}^{(i)} = \arg \max\limits_{v_i \in [v_i]} y_j$, $\underline{y_j}^{(i)} = \arg \min\limits_{v_i \in [v_i]} y_j$.

If the condition (10) is not satisfied, then the interval Δ_i is for y_j considered to be insignificant, and while determining $\underline{y_j}$, $\overline{y_j}$ it is given in the form of a point with a value $\mathrm{mid}[v_i]$, $i \in N_j$. Set N_j is a set of indices *i*, defining the parameters $[v_i]$ which can be set as a number $\mathrm{mid}[v_i]$ in calculation of $\underline{y_j}$ and $\overline{y_j}$. Thus, parameters v_i for $i \in N_j$ are excluded from the number of varying parameters in solving (5), (6) to determine the lower and upper boundaries $\underline{y_j}$, $\overline{y_j}$.

Due to the above, the formulations (5), (6) can be represented as:

$$[y_j] = \begin{bmatrix} \min\limits_{\substack{v_i \in [v_i] \\ \forall i \in G_j}} \left\{ y_j \middle| \mathrm{M}(y, u, x, v_i, \mathrm{mid}[v_l], \underline{\hat{v}}_k) = 0 \right\} \\ \max\limits_{\substack{v_i \in [v_i] \\ \forall i \in G_j}} \left\{ y_j \middle| \mathrm{M}(y, u, x, v_i, \mathrm{mid}[v_l], \overline{\hat{v}}_k) = 0 \right\} \end{bmatrix}, \tag{11}$$

$$G_j = F \setminus (N_j \cup L_j),$$

$$l \in N_j, \; k \in L_j, \; j = \overline{1, m}.$$

3. The method for solution of interval static model with distributed parameters

Another class of mathematical models includes interval static models with distributed parameters, which are defined by equation

$$\forall v \in [v]: \mathrm{M}(y'(z), y(z), u, x, v, z) = 0, \tag{12}$$

where the *z* – is space object coordinate.

In the first phase of the study of dependence $y_j(z)=y_j(z)(v_i)$, $i=\overline{1,p}$, $j=\overline{1,m}$, $z\in[0,Z]$. The rule of calculating the boundaries $\underline{y_j}(z)$, $\overline{y_j}(z)$ is determined. In the course of the study (Fig. 3) for each $v_{i(k)}$ $\left(v_{i(k)_i}\in[v_i],\ k=\overline{1,K_i},\ v_{i(k+1)}-v_{i(k)}=\Delta h\right)$ for the dependencies $y_{kj}(z)=y_{kj}(z)\left(v_{i(k)}\right)$ are constructed.

Further, on the spatial coordinate z of the object with a step Δz points $z_1, z_2, \ldots, z_s, \ldots, z_S$ are defined. It results in the sequence

$$\left\{y_{11j}^{(i)}, y_{21j}^{(i)}, \ldots, y_{k1j}^{(i)}, \ldots, y_{K_i1j}^{(i)}\right\}, \ldots, \left\{y_{1sj}^{(i)}, y_{2sj}^{(i)}, \ldots, y_{ksj}^{(i)}, \ldots, y_{K_isj}^{(i)}\right\}, \ldots,$$

$$\left\{y_{1Sj}^{(i)}, y_{2Sj}^{(i)}, \ldots, y_{kSj}^{(i)}, \ldots, y_{K_iSj}^{(i)}\right\}, \tag{13}$$

where the first subindex means the number of the curve (in Fig. 3, the curve $y_{kj}(z)$ is denoted as curve 1, curve 2, ..., curve k, ...,); the second subindex corresponds to the line number, denoted as the cross section s, $s=\overline{1,S}$, on which the point $y_{ksj}^{(i)}$, is and the third index is the number of the output parameter y_j.

If for a given i, all S sequences (13) are monotonic, then $i\in L_j$ (Fig. 4, a). In this case, the lower and upper boundaries are defined by:

$$\underline{y_j}(z)^{(i)}=y_j(z)\left(\underline{\hat{v}}_i\right), \tag{14}$$

$$\overline{y_j}(z)^{(i)}=y_j(z)\left(\overline{\hat{v}}_i\right). \tag{15}$$

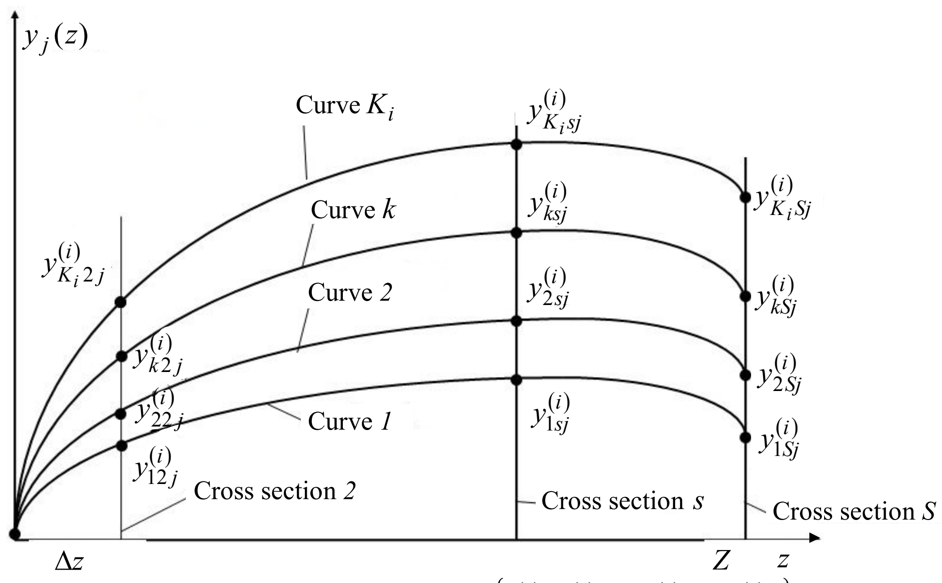


Fig. 3. Construction of sequences $\left\{y_{1sj}^{(i)}, y_{2sj}^{(i)}, \ldots, y_{ksj}^{(i)}, \ldots, y_{K_isj}^{(i)}\right\}$ in the study of dependence $y_j(z)^{(i)}$

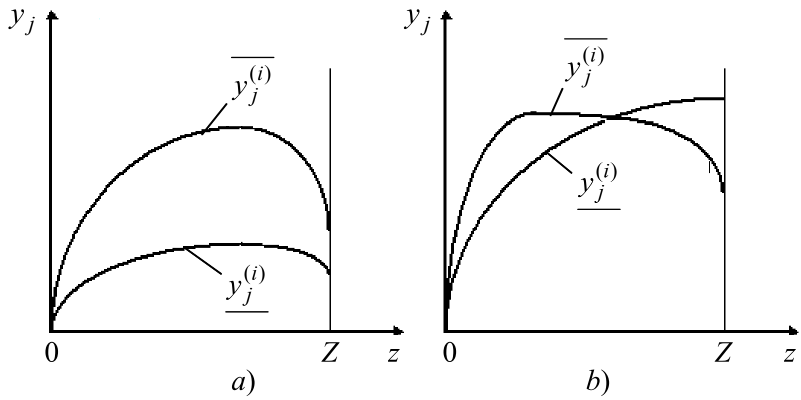


Fig. 4. Definition of the boundaries $[y_j(z)]$ of the interval for the dependence $y_j(z)^{(i)} = y_j(z)(v_i)\ (v_i \in [v_i])$ in $i \in L_j$ (*a*), with $i \notin L_j$ (*b*)

If at least one sequence of s, $s = \overline{1, S}$, is nonmonotonic, then $i \notin L_j$ (Fig. 4, *b*).

At the second stage the value of the interval Δ_i is determined for the output variable $y_j(z)^{(i)}$, according to the inequality:

$$\max_z \left| \overline{y_j}(z)^{(i)} - \underline{y_j}(z)^{(i)} \right| \geq \varepsilon_j^i. \tag{16}$$

Here $\overline{y_j}(z)^{(i)}$, $\underline{y_j}(z)^{(i)}$ are determined from (14), (15) if $i \in L_j$, otherwise, from the solutions of optimization problems $\overline{y_j}(z)^{(i)} = \arg \max\limits_{v_i \in [v_i]} y_j(z)$, $\underline{y_j}(z)^{(i)} = \arg \min\limits_{v_i \in [v_i]} y_j(z)$.

If the condition (16) is not satisfied, then the interval Δ_i for $y_j(z)$ is considered to be insignificant, and in determining $\underline{y_j}(z)$, $\overline{y_j}(z)$ is given in the form of a point with a value $\mathrm{mid}[v_i]$, $i \in N_j$. Parameters v_i for $i \in N_j$ are excluded from the number of varying parameters in solving (5), (6) to determine the lower and upper boundaries $\underline{y_j}(z)$, $\overline{y_j}(z)$.

As a result of research interval output parameter $[y_j(z)]$ is defined by:

$$[y_j(z)] = \begin{bmatrix} \min\limits_{\substack{v_i \in [v_i] \\ \forall i \in G_j}} \left\{ y_j(z) \middle| \mathrm{M}\left(y'(z), y(z), u, x, v_i, \mathrm{mid}[v_l], \underline{\hat{v}_k}, z\right) = 0 \right\}, \\ \max\limits_{\substack{v_i \in [v_i] \\ \forall i \in G_j}} \left\{ y_j(z) \middle| \mathrm{M}\left(y'(z), y(z), u, x, v_i, \mathrm{mid}[v_l], \hat{\overline{v}}_k, z\right) = 0 \right\} \end{bmatrix}, \tag{17}$$

$$z \in [0, Z], \quad G_j = F \setminus (N_j \cup L_j),$$
$$l \in N_j, \quad k \in L_j, \quad j = \overline{1, m}.$$

If interval dynamic model with lumped parameters is used as a mathematical model

$$\forall v \in [v]: \mathrm{M}(y'(\tau), y(\tau), u, x, v, \tau) = 0 \tag{18}$$

then method of studying model (18) is similar to the method used for model (12). In this case, the coordinate z is by time parameter τ.

4. The method for solution of interval dynamic model with distributed parameters

Research for interval dynamic model with distributed parameters:

$$\forall v \in [v]: \mathrm{M}(y'_\tau(\tau, z) y'_z(\tau, z), u, x, v, \tau, z) = 0, \tag{19}$$

where $y'_\tau(\tau, z) y'_z(\tau, z)$ – are partial derivatives, with time and coordinate respectively, is based on the methodology applied to model (12).

At the first stage, like in the procedure for model (12), a study of $y_j(\tau, z)(v_i)$ is conducted to define membership of i to set L_j.

First, for a fixed time $\tau = \tau_c$ for $z \in [0, Z]$ the analysis of interval parameter $[v_i]$ is carried out to define the membership of i to set L_j. Then in point $z = z_c$ for $\tau \in [0, T]$ a similar study is carried out.

If in both cases, the ratio $i \in L_j$ is true, then in calculating $[y_j]$ only upper $\overline{v_i}$ and lower $\underline{v_i}$ boundaries of parameter $[v_i]$ should be taken into account, that is parameter $[v_i]$ corresponds to the set L_j.

At the second stage the analysis of the significance of the interval Δ_i is carried out. If the interval Δ_i is insignificant simultaneously for the dependence of $y_j(\tau_c, z)$, with $\tau \in [0, T]$ and $y_j(\tau, z_c)$ at $z \in [0, Z]$, then parameter $[v_i]$, can be replaced point by $\mathrm{mid}[v_i]$ and $i \in N_j$ be taken.

As a result of studies interval output parameter of the model (19) is determined by:

$$[y_j(\tau, z)] = \begin{bmatrix} \min\limits_{\substack{v_i \in [v_i] \\ \forall i \in G_j}} \left\{ y_j(\tau, z) \middle| \mathrm{M}\left(y'_\tau(\tau, z) y'_z(\tau, z), u, x, v_i, \mathrm{mid}[v_l], \underline{v_k}, \tau, z\right) = 0 \right\} \\ \max\limits_{\substack{v_i \in [v_i] \\ \forall i \in G_j}} \left\{ y_j(\tau, z) \middle| \mathrm{M}\left(y'_\tau(\tau, z) y'_z(\tau, z), u, x, v_i, \mathrm{mid}[v_l], \hat{\bar{v}}_k, \tau, z\right) = 0 \right\} \end{bmatrix}, \tag{20}$$

$$z \in [0, Z], \; \tau \in [0, T],$$

$$G_j = F \setminus (N_j \cup L_j), \quad l \in N_j, \quad k \in L_j, \quad j = \overline{1, m}.$$

Thus, the method proposed allows finding the output parameters of an interval model, which are defined by their upper and lower boundaries.

5. Study of the firing process of the material in a rotating furnace, based on an interval model

This method has been implemented in the firing process in a rotary furnace taken as an example. Rotary furnace is a cylindrical industrial furnace with a rotational movement around its longitudinal axis, designed for heating or burning materials with the aim of their physic-chemical treatment. To maintain the temperature flaring of natural gas or fuel oil is applied. The material in the furnace moves counter flow to the products of combustion.

We have developed a mathematical model of the firing process in a rotary furnace:

$$\frac{dT_{\mathrm{m}}}{dl}=\frac{\varepsilon_{\mathrm{m}}\sigma_0\left(T_{\mathrm{g}}^4-T_{\mathrm{m}}^4\right)\pi d\Delta l-\dfrac{2\pi(T_{\mathrm{m}}-T_{\mathrm{w}})\Delta l}{\dfrac{1}{\lambda}\ln\dfrac{D}{d}}}{\theta_{\mathrm{m}}G_{\mathrm{m}}};\tag{21}$$

$$\frac{dT_{\mathrm{g}}}{dl}=\frac{\varepsilon_{\mathrm{m}}\sigma_0\left(T_g^4-T_m^4\right)\pi d\Delta l-2mG_f\Omega \mathrm{e}^{-m(L-l)^2}(L-l)}{\theta_g G_{\mathrm{g}}};\tag{22}$$

$$\frac{2\pi(T_{\mathrm{m}}-T_{\mathrm{w}})\Delta l}{\dfrac{1}{\lambda}\ln\dfrac{D}{d}}=\varepsilon_{\mathrm{w}}\sigma_0\left(T_{\mathrm{w}}^4-T_{\mathrm{am}}^4\right)\pi D\Delta l;\tag{23}$$

$$T_{\mathrm{m}}\,|_{l=0}=T_{\mathrm{m}}^{\mathrm{en}};\tag{24}$$

$$T_{\mathrm{g}}\,|_{l=L}=T_{\mathrm{g}}^{\mathrm{ex}},\tag{25}$$

where T_{m} – temperature of the material, K; T_{g} – gas temperature, K; T_{w} – wall temperature, K; T_{am} – ambient temperature, K; l – the current length of the furnace, m; ε_{w} – the emissivity of the lining; ε_{m} – the emissivity of the material; d – the inner furnace diameter, m; D – outer furnace diameter, m; L – the total length of the furnace, m; σ_0 – ratio of blackbody radiation, $\mathrm{W/(m^2{\cdot}K^4)}$; λ – thermal conductivity of the lining material, W/(m·K); Ω – the heat from 1 kg of fuel combustion (specific heat of combustion), J/kg; θ_{m} – heat capacity of the material, J/(kg·°C); θ_{g} – gas heat capacity, J/(kg·°C); G_{m} – material consumption, kg/s; G_{g} – gas consumption, $\mathrm{m^3/s}$; G_{f} – fuel consumption, $\mathrm{m^3/s}$; m – empirical coefficient.

This mathematical model has uncertain parameters. This is the emissivity of the material ε_{m}, the heat capacity of the material θ_{m} and the length of the torch l_{t}. Empirical coefficient m depends on the length of the torch, thus changing the length of the flare leads to a change in the empirical coefficient.

A mathematical model of the firing process in a rotary furnace belongs to a class of static models with distributed parameters (12).

Uncertain parameters of the mathematical model are given in the form of interval numbers: $[\varepsilon_{\mathrm{m}}]$ = [0.3, 0.7]; $[\theta_{\mathrm{m}}]$ = [1000, 1200]; $[l_{\mathrm{t}}]$ = [7, 15]. In the mathematical model the output parameters are: temperature distribution of gas,

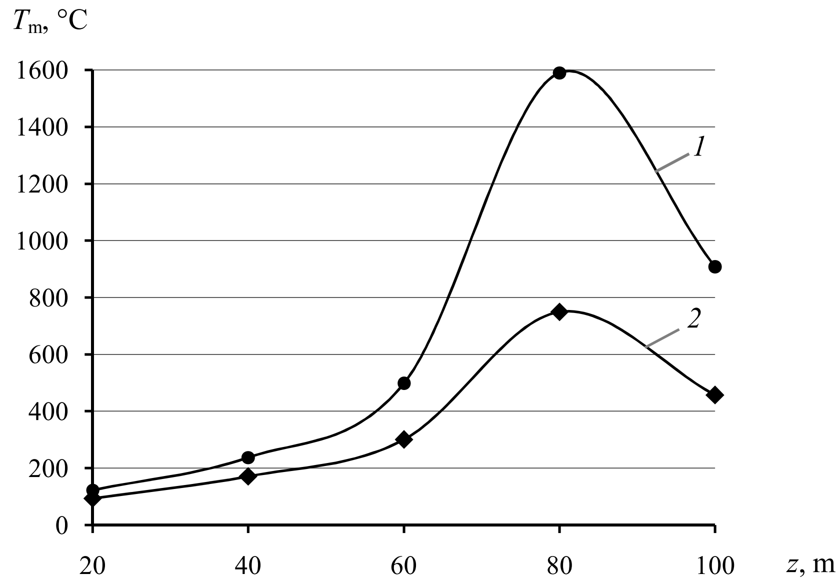


Fig. 5. The boundaries of the interval $[y_j(z)]$:

1 – upper boundary; *2* – lower boundary

material and wall along the length of the furnace. The most important parameter for the firing process is the temperature of the material; therefore, all calculations have been performed only for the temperature of the material.

To solve the model is to calculate the upper and lower boundaries of the entire interval $[y_j(z)]$ of the temperature distribution of the material (Fig. 5).

Then, the significance of the interval Δ_i is defined for the output variable, according to (16). Parameter ε_j^i is a tolerable error, the value of which is given by a technologist. In this case, the error is 3 °C. As can be seen from Fig. 5 interval Δ_i is significant. The uncertainty of the initial data of a model is essential. To solve the problems you need to specify the initial data.

The proposed method allows to calculate the interval of output values of mathematical models, which makes it advantageous over other methods of disclosing the uncertainty in mathematical models.

Conclusion

The article argues that effective methods of presenting the undetermined parameters of mathematical models are interval numbers. The algorithms for solving the interval mathematical models of statics and dynamics with lumped and distributed parameters. The algorithms are based on solving optimization problems. The implementation of the algorithms is exemplified by the model of the firing process of the material in a rotary furnace, in which the information about some of the parameters is given in the form of interval numbers. Interval mathematical models can be used at the preliminary stage of research systems, as well as when it is needed obtain guaranteed results if the parameters are unknown.

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Разработка интервальных математических моделей процессов теплообмена

Т.А. Фролова, С.В. Фролов, Д.С. Туляков

ФГБОУ ВПО «Тамбовский государственный технический университет», г. Тамбов

Ключевые слова и фразы: вращающаяся печь; интервальные математические модели; интервальные числа; обжиг.

Аннотация: Показано, что эффективным методом представления неопределенных параметров математических моделей являются интервальные числа. Предложены алгоритмы решения интервальных математических моделей статики и динамики с сосредоточенными и распределенными параметрами. Алгоритмы основаны на решении оптимизационных задач. Реализация представленных алгоритмов продемонстрирована на примере модели процесса обжига материала во вращающейся печи, где информация о некоторых параметрах задается в виде интервальных чисел. Представленная методика позволяет эффективно решать задачи проектирования и управления технологическими процессами в условиях неопределенности.
