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EXACT SOLUTION FOR FUNCTIONALLY GRADED PIEZOELECTRIC CYLINDRICAL SHELLS

G.M. Kulikov[1], A.V. Erofeev[2]

Departments: "Applied Mathematics and Mechanics" (1), "Construction of Buildings and Structures" (2), TSTU; kulikov@apmath.tstu.ru

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Abstract: This paper presents a sampling surfaces method applied to functionally graded piezoelectric cylindrical shells. It is shown that the sampling surfaces method can be utilized efficiently to derivation of 3D exact solutions for functionally graded piezoelectric cylindrical shells with a specified accuracy by using the sufficiently large number of sampling surfaces located at Chebyshev polynomial nodes.

1. Introduction

Nowadays, the functionally graded (**FG**) piezoelectric materials are widely used in mechanical engineering due to their advantages compared to traditional piezoelectric materials [1]. At the same time, the study of FG piezoelectric structures is not a simple task [2] because the material properties depend on the transverse coordinate and some specific assumptions concerning their variations in the thickness direction are required (see, e.g. [3]). In practice, this implies that we deal here with a system of differential equations with variable coefficients. Therefore, the conventional approaches can not be applied directly to 3D exact solutions for FG piezoelectric shells. However, it is possible if the shell is artificially divided into a large number of individual layers with equal thicknesses [4]. As a matter of fact, the use of such a technique means that the solutions derived are just approximate [5, 6]. On the contrary, the asymptotic approach to 3D solutions for FG piezoelectric plates and shells [7, 8] yields exact results because governing differential equations are obtained through definite integration in the thickness direction.

The present paper is intended to show that the sampling surfaces (**SaS**) method [9] can be also applied efficiently to 3D exact solutions of electroelasticity for FG piezoelectric cylindrical shells. In accordance with this method, we choose inside the shell body N not equally spaced SaS $\Omega^1, \Omega^2, ..., \Omega^N$ parallel to the middle surface of the shell and introduce displacement vectors $\mathbf{u}^1, \mathbf{u}^2, ..., \mathbf{u}^N$ and electric potentials $\varphi^1, \varphi^2, ..., \varphi^N$ of these surfaces as basic shell variables, where $N \geq 3$. Such choice of unknowns in conjunction with the use of Lagrange polynomials of degree $N-1$ in the thickness direction permits one, first, to represent governing equations of the proposed FG shell formulation in a very compact form and, second, to adopt strain-displacement equations, which describe exactly all rigid-body shell motions in any convected curvilinear coordinate system [10, 11]. Note also that the SaS method has been already applied to the 3D exact analysis of elastic and piezoelectric plates and shells [12–16].

It should be mentioned that the developed approach with equally spaced SaS [9] does not work properly with Lagrange polynomials of high degree because the Runge's phenomenon can occur, which yields the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of equally spaced nodes is increased then the oscillations become even larger. However, the use of Chebyshev polynomial nodes [12] can help to improve significantly the behaviour of Lagrange polynomials of high degree for which the error will go to zero as $N \to \infty$. This fact gives an opportunity to derive the 3D exact solutions for FG piezoelectric cylindrical shells with a prescribed accuracy employing the sufficient number of SaS located at Chebyshev polynomial nodes.

2. Kinematic description of shell

Consider a shell of the thickness *h*. Let the middle surface Ω be described by orthogonal curvilinear coordinates θ_1 and θ_2, which are referred to the lines of principal curvatures of its surface. The coordinate θ_3 is oriented along the unit vector $\mathbf{e}_3$ normal to the middle surface. Introduce the following notations: $\mathbf{e}_\alpha$ are the orthonormal base vectors; A_α are the coefficients of the first fundamental form; k_α are the principal curvatures of the middle surface; $c_\alpha = 1 + k_\alpha \theta_3$ are the components of the shifter tensor; $c_\alpha^I = 1 + k_\alpha \theta_3^I$ are the components of the shifter tensor at SaS Ω^I (Fig. 1); θ_3^I are the transverse coordinates of SaS located at Chebyshev polynomial nodes [17]

$$\theta_3^I = -\frac{h}{2}\cos\left(\pi \frac{2I-1}{2N}\right). \tag{1}$$

Here and in the following developments, the index *I* and the indices *J*, *K* to be introduced later identify the belonging of any quantity to the SaS and take values $1, 2, ..., N$; Greek indices α, β range from 1 to 2; Latin tensorial indices i, j, k, l range from 1 to 3.

The use of notations

$$\mathbf{u}^I = \mathbf{u}(\theta_3^I); \quad \boldsymbol{\beta}^I = \mathbf{u}_{,3}(\theta_3^I) \tag{2}$$

and strain-displacement relationships [9] yields

$$2\varepsilon_{\alpha\beta}^I = \frac{1}{A_\alpha c_\alpha^I}\mathbf{u}_{,\alpha}^I \mathbf{e}_\beta + \frac{1}{A_\beta c_\beta^I}\mathbf{u}_{,\beta}^I \mathbf{e}_\alpha;$$

$$2\varepsilon_{\alpha 3}^I = \boldsymbol{\beta}^I \mathbf{e}_\alpha + \frac{1}{A_\alpha c_\alpha^I}\mathbf{u}_{,\alpha}^I \mathbf{e}_3; \quad \varepsilon_{33}^I = \boldsymbol{\beta}^I \mathbf{e}_3, \tag{3}$$

where $\mathbf{u}$ is the displacement vector; $\mathbf{u}^I(\theta_1, \theta_2)$ are the displacement vectors of SaS; $\boldsymbol{\beta}^I(\theta_1, \theta_2)$ are the values of the derivative of the displacement vector with respect to coordinate θ_3 at SaS; $(\ldots)_{,i}$ stands for the partial derivatives with respect to coordinates θ_i.

Next, we represent the displacement vectors $\mathbf{u}^I$ and $\boldsymbol{\beta}^I$ in the reference surface frame $\mathbf{e}_i$ as follows:

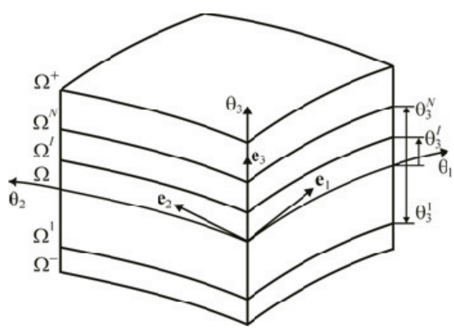

Fig. 1. Geometry of the shell

$$\mathbf{u}^I = u_i^I \mathbf{e}_i; \quad \boldsymbol{\beta}^I = \beta_i^I \mathbf{e}_i. \tag{4}$$

Using (4) and well-known formulas for the derivatives of unit vectors $\mathbf{e}_i$ with respect to orthogonal curvilinear coordinates θ_α (see, e.g. [12, 14]), one obtains

$$\frac{1}{A_\alpha}\mathbf{u}^I_{,\alpha} = \lambda^I_{i\alpha}\mathbf{e}_i, \tag{5}$$

where

$$\lambda^I_{\alpha\alpha} = \frac{1}{A_\alpha}u^I_{\alpha,\alpha} + B_\alpha u^I_\beta + k_\alpha u^I_3, \quad \lambda^I_{\beta\alpha} = \frac{1}{A_\alpha}u^I_{\beta,\alpha} - B_\alpha u^I_\alpha \quad (\beta \neq \alpha),$$

$$\lambda^I_{3\alpha} = \frac{1}{A_\alpha}u^I_{3,\alpha} - k_\alpha u^I_\alpha, \quad B_\alpha = \frac{1}{A_\alpha A_\beta}A_{\alpha,\beta} \quad (\beta \neq \alpha). \tag{6}$$

Substituting (4) and (5) in strain-displacement relationships (3) and accounting for the orthogonality of unit vectors $\mathbf{e}_i$, we obtain

$$2\varepsilon^I_{\alpha\beta} = \frac{1}{c^I_\beta}\lambda^I_{\alpha\beta} + \frac{1}{c^I_\alpha}\lambda^I_{\beta\alpha};$$

$$2\varepsilon^I_{\alpha 3} = \beta^I_\alpha + \frac{1}{c^I_\alpha}\lambda^I_{3\alpha}; \qquad \varepsilon^I_{33} = \beta^I_3. \tag{7}$$

Up to this moment, no assumptions concerning displacement and strain fields have been made. We start now with the first fundamental assumption of the proposed FG piezoelectric shell formulation. Assume that the displacements are distributed through the thickness of the shell as follows

$$u_i = \sum_I L^I u_i^I, \tag{8}$$

where $L^I(\theta_3)$ are the Lagrange polynomials of degree $N-1$ expressed as

$$L^I = \prod_{J \neq I}\frac{\theta_3 - \theta_3^J}{\theta_3^I - \theta_3^J} \tag{9}$$

such that $L^I\left(\theta_3^J\right) = 1$ for $J = I$ and $L^I\left(\theta_3^J\right) = 0$ for $J \neq I$.

The use of relations (2), (4) and (8) leads to

$$\beta_i^I = \sum_J M^J(\theta_3^I)u_i^J, \tag{10}$$

where $M^I = L^I_{,3}$ are the polynomials of degree $N-2$. Their values on SaS can be written as

$$M^J\left(\theta_3^I\right) = \frac{1}{\theta_3^J - \theta_3^I}\prod_{K \neq I,J}\frac{\theta_3^I - \theta_3^K}{\theta_3^J - \theta_3^K} \quad (J \neq I);$$

$$M^I\left(\theta_3^I\right) = -\sum_{J \neq I} M^J\left(\theta_3^I\right). \tag{11}$$

Thus, the key functions β_i^I of the proposed FG shell formulation are represented according to (11) as a linear combination of displacements of SaS u_i^J.

The following step consists in a choice of the correct approximation of strains through the thickness of the shell. It is apparent that the strain distribution should be chosen similar to the displacement distribution (8). Therefore, the second fundamental assumption of the FG shell formulation can be written as

$$\varepsilon_{ij} = \sum_I L^I \varepsilon_{ij}^I. \tag{12}$$

Remark 1. Strain-displacement relationships (7) and (12) exactly represent all rigid-body motions of a shell in any convected curvilinear coordinate system. The proof of this statement can be done invoking the results [14]. Note also that the origins of using the strains of SaS can be found in contributions [18, 19].

3. Description of electric field

The relation between the electric field and the electric potential φ is given by

$$E_i = -\frac{1}{A_i c_i}\varphi_{,i}. \tag{13}$$

In particular, the electric field vector at SaS is presented as

$$E_\alpha^I = E_\alpha(\theta_3^I) = -\frac{1}{A_\alpha c_\alpha^I}\varphi_{,\alpha}^I; \tag{14}$$

$$E_3^I = E_3(\theta_3^I) = -\psi^I, \tag{15}$$

where $\varphi^I(\theta_1,\theta_2)$ are the electric potentials of SaS; $\psi^I(\theta_1,\theta_2)$ are the values of the derivative of the electric potential with respect to thickness coordinate θ_3 at SaS, that is,

$$\varphi^I = \varphi(\theta_3^I); \quad \psi^I = \varphi_{,3}(\theta_3^I). \tag{16}$$

Now, we accept the third and fourth fundamental assumption*s* of the proposed FG piezoelectric shell formulation concerning the distributions of the electric potential and the electric field vector through the shell thickness, that is,

$$\varphi = \sum_I L^I \varphi^I; \tag{17}$$

$$E_i = \sum_I L^I E_i^I. \tag{18}$$

The use of (16) and (17) leads to a simple formula

$$\psi^I = \sum_J M^J(\theta_3^I)\varphi^J, \tag{19}$$

which is similar to (10). This implies that the key functions ψ^I of the FG piezoelectric shell formulation are represented as a linear combination of electric potentials of SaS φ^J.

4. Variational formulation

The variational equation for the FG piezoelectric shell can be written as

$$\delta\Pi = 0, \tag{20}$$

where Π is the extended potential energy [20] of the shell defined as

$$\Pi = \frac{1}{2}\iint_\Omega \int_{-h/2}^{h/2} \left(\sigma_{ij}\varepsilon_{ij} - D_i E_i\right) A_1 A_2 c_1 c_2 d\theta_1 d\theta_2 d\theta_3 - W; \tag{21}$$

$$W = \iint_{\Omega} \left(p_i^+ u_i^+ + q^+ \varphi^+ \right) A_1 A_2 c_1^+ c_2^+ d\theta_1 d\theta_2 -$$
$$- \iint_{\Omega} \left(p_i^- u_i^- + q^- \varphi^- \right) A_1 A_2 c_1^- c_2^- d\theta_1 d\theta_2 + W_\Sigma , \tag{22}$$

where σ_{ij} is the stress tensor; D_i is the electric displacement vector; $u_i^- = u_i(-h/2)$ and $u_i^+ = u_i(h/2)$ are the displacements of bottom and top surfaces Ω^- and Ω^+; $\varphi^- = \varphi(-h/2)$ and $\varphi^+ = \varphi(h/2)$ are the electric potentials of bottom and top surfaces; $c_\alpha^- = 1 - k_\alpha h/2$ and $c_\alpha^+ = 1 + k_\alpha h/2$ are the components of the shifter tensor on outer surfaces; p_i^- and p_i^+ are the loads acting on outer surfaces; q^- and q^+ are the electric charges on outer surfaces; W_Σ is the work done by external electromechanical loads applied to the boundary surface Σ.

Substituting strain and electric field distributions (12), (17) in (21) and introducing stress resultants

$$H_{ij}^I = \int_{-h/2}^{h/2} \sigma_{ij} L^I c_1 c_2 d\theta_3 \tag{23}$$

and electric displacement resultants

$$T_i^I = \int_{-h/2}^{h/2} D_i L^I c_1 c_2 d\theta_3 , \tag{24}$$

one obtains

$$\Pi = \frac{1}{2} \iint_{\Omega} \sum_I \left(H_{ij}^I \varepsilon_{ij}^I - T_i^I E_i^I \right) A_1 A_2 d\theta_1 d\theta_2 - W . \tag{25}$$

For simplicity, we consider the case of linear piezoelectric materials described as

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} - e_{kij} E_k ; \tag{26}$$

$$D_i = e_{ikl} \varepsilon_{kl} + \in_{ik} E_k , \tag{27}$$

where C_{ijkl}, e_{ikl} and $\in_{ik}$ are the elastic, piezoelectric and dielectric constants of the material.

Finally, we accept the last fifth fundamental assumption of the FG piezoelectric shell formulation. Let us assume that the material constants are distributed through the thickness of the shell according to the following law:

$$C_{ijkl} = \sum_I L^I C_{ijkl}^I ; \quad e_{ikl} = \sum_I L^I e_{ikl}^I ; \quad \in_{ik} = \sum_I L^I \in_{ik}^I \tag{28}$$

that is extensively utilized in this paper. Here, C_{ijkl}^I, e_{ikl}^I and $\in_{ik}^I$ are the values of elastic, piezoelectric and dielectric constants on SaS.

Inserting constitutive equations (26), (27) correspondingly in (23), (24) and using the through-thickness distributions (12), (18) and (28), we arrive at required formulas for stress and electric displacement resultants:

$$H_{ij}^I = \sum_{J,K} \Lambda^{IJK} \left(C_{ijkl}^J \varepsilon_{kl}^K - e_{kij}^J E_k^K \right); \tag{29}$$

$$T_i^I = \sum_{J,K} \Lambda^{IJK} \left(e_{ikl}^J \varepsilon_{kl}^K + \in_{ik}^J E_k^K \right), \tag{30}$$

where

$$\Lambda^{IJK} = \int_{-h/2}^{h/2} L^I L^J L^K c_1 c_2 d\theta_3. \tag{31}$$

5. Exact solution for FG piezoelectric cylindrical shells

Consider a FG piezoelectric cylindrical shell and assume that its middle surface is described by axial and circumferential coordinates θ_1 and θ_2. The boundary conditions for the simply supported cylindrical shell with electrically grounded edges are written as follows:

$$\sigma_{11} = u_2 = u_3 = \varphi = 0 \text{ at } \theta_1 = 0 \text{ and } \theta_1 = L, \tag{32}$$

where L is the length of the shell. To satisfy boundary conditions (32), we search the analytical solution of the problem by a method of double Fourier series expansion:

$$u_1^I = \sum_{r=1}^{\infty}\sum_{s=0}^{\infty} u_{1rs}^I \cos\frac{r\pi\theta_1}{L}\cos s\theta_2; \quad u_2^I = \sum_{r=1}^{\infty}\sum_{s=0}^{\infty} u_{2rs}^I \sin\frac{r\pi\theta_1}{L}\sin s\theta_2;$$

$$u_3^I = \sum_{r=1}^{\infty}\sum_{s=0}^{\infty} u_{3rs}^I \sin\frac{r\pi\theta_1}{L}\cos s\theta_2; \quad \varphi^I = \sum_{r=1}^{\infty}\sum_{s=0}^{\infty} \varphi_{rs}^I \sin\frac{r\pi\theta_1}{L}\cos s\theta_2. \tag{33}$$

The external electromechanical loads are also expanded in double Fourier series.

Substituting (33) and Fourier series corresponding to electromechanical loading in the extended potential energy (22), (25) with $W_\Sigma = 0$ and using relations (6), (7), (10), (14), (15), (19), (29), (30) and (31), one finds

$$\Pi = \sum_{r=1}^{\infty}\sum_{s=0}^{\infty} \Pi_{rs}(u_{irs}^I, \varphi_{rs}^I). \tag{34}$$

Invoking the variational equation (20) and taking into account (34), the following system of linear algebraic equations of order $4N$ is obtained:

$$\frac{\partial \Pi_{rs}}{\partial u_{irs}^I} = 0; \quad \frac{\partial \Pi_{rs}}{\partial \varphi_{rs}^I} = 0. \tag{35}$$

The linear system (35) is solved using a method of Gaussian elimination.

The described algorithm was performed with the Symbolic Math Toolbox, which incorporates symbolic computations into the numeric environment of MATLAB. This allows the derivation of exact solutions of 3D electroelasticity for FG piezoelectric cylindrical shells with a specified accuracy.

As a numerical example, we study a FG piezoelectric cylindrical shell subjected to mechanical loading acting on the top surface:

$$\sigma_{33}^+ = p_0 \sin\frac{\pi\theta_1}{L}\cos\theta_2; \quad \sigma_{13}^- = \sigma_{13}^+ = \sigma_{23}^- = \sigma_{23}^+ = \sigma_{33}^- = D_3^- = D_3^+ = 0, \tag{36}$$

or electric loading acting on the same surface

$$D_3^+ = q_0 \sin\frac{\pi\theta_1}{L}\cos\theta_2; \quad \sigma_{13}^- = \sigma_{13}^+ = \sigma_{23}^- = \sigma_{23}^+ = \sigma_{33}^- = \sigma_{33}^+ = D_3^- = 0, \tag{37}$$

where $p_0 = 1$ Pa and $q_0 = 10^{-8}$ $\mathrm{C/m^2}$.

Here, we consider and compare two basic approaches [2] widely used for describing the FG piezoelectric materials. The first approach known in the literature as an exponential law can be presented in the following form:

$$C_{ijkl} = C_{ijkl}^{-} e^{\alpha(z+0,5)}; \quad e_{ikl} = e_{ikl}^{-} e^{\alpha(z+0,5)}; \quad \in_{ik} = \in_{ik}^{-} e^{\alpha(z+0,5)}; \quad z = \theta_3 / h, \tag{38}$$

where C_{ijkl}^{-}, e_{ikl}^{-} and $\in_{ik}^{-}$ are the values of elastic, piezoelectric and dielectric constants on the bottom surface; α is the material gradient index given by

$$\alpha = \ln \frac{C_{ijkl}^{+}}{C_{ijkl}^{-}} = \ln \frac{e_{ikl}^{+}}{e_{ikl}^{-}} = \ln \frac{\in_{ik}^{+}}{\in_{ik}^{+}}, \tag{39}$$

where C_{ijkl}^{+}, e_{ikl}^{+} and $\in_{ik}^{+}$ are the values of elastic, piezoelectric and dielectric constants on the top surface. Next, we study the most popular power law. The latter law reflects a simple rule of mixtures efficiently utilized for finding the effective properties of the FG piezoelectric material, that is,

$$C_{ijkl} = C_{ijkl}^{-} V^{-} + C_{ijkl}^{+} V^{+}; \quad e_{ikl} = e_{ikl}^{-} V^{-} + e_{ikl}^{+} V^{+};$$

$$\in_{ik} = \in_{ik}^{-} V^{-} + \in_{ik}^{+} V^{+}; \quad V^{+} = 1 - V^{-}, \quad V^{-} = (0,5 - z)^{\beta}, \tag{40}$$

where $V^{-}(z)$ is the volume fraction; β is the material gradient index.

The material constants on the bottom surface are considered to be the same as those of the PZT-4 [3]:

$$C_{1111} = C_{2222} = 139{,}0\,\text{GPa}, \quad C_{3333} = 115{,}0\,\text{GPa}, \quad C_{1122} = 77{,}8\,\text{GPa},$$

$$C_{1133} = C_{2233} = 74{,}3\,\text{GPa}, \quad C_{2323} = C_{1313} = 25{,}6\,\text{GPa}, \quad C_{1212} = 30{,}6\,\text{GPa},$$

$$e_{311} = e_{322} = -5{,}2\,\text{C/m}^2, \quad e_{333} = 15{,}1\,\text{C/m}^2, \quad e_{113} = e_{223} = 12{,}7\,\text{C/m}^2,$$

$$\in_{11} = \in_{22} = 13{,}06\,\text{nF/m}, \quad \in_{33} = 11{,}51\,\text{nF/m},$$

whereas the material constants on the top surface are three times more than those of the PZT-4. To investigate the response of the FG piezoelectric cylindrical shell more carefully, we consider four values of the material gradient index: $\alpha = 1{,}0986$ in the case of using the exponential law (38), i.e. only one value can be chosen according to (39), and $\beta = 0{,}1; 2; 10$ in the case of the power law (40), which allows many values to be taken as illustrated in Fig. 2.

The geometric parameters of the shell are taken as $L = 4\,\text{m}$, $R = 1\,\text{m}$ and $h = 0{,}1\,\text{m}$, where R is the radius of the middle cylindrical surface. To analyze the derived results for both types of loading (36) and (37) effectively, we introduce the following scaled field variables at crucial points:

$$\bar{u}_3 = 10^{11} u_3(L/2, 0, z);$$

$$\bar{\sigma}_{11} = \sigma_{11}(L/2, 0, z);$$

$$\bar{\sigma}_{12} = \sigma_{12}(0, \pi/2, z);$$

$$\bar{\sigma}_{13} = 10^2 \sigma_{13}(0, 0, z);$$

$$\bar{\sigma}_{23} = 10^2 \sigma_{23}(L/2, \pi/2, z);$$

$$\bar{\sigma}_{33} = \sigma_{33}(L/2, 0, z);$$

$$\bar{\varphi} = 10^3 \varphi(L/2, 0, z);$$

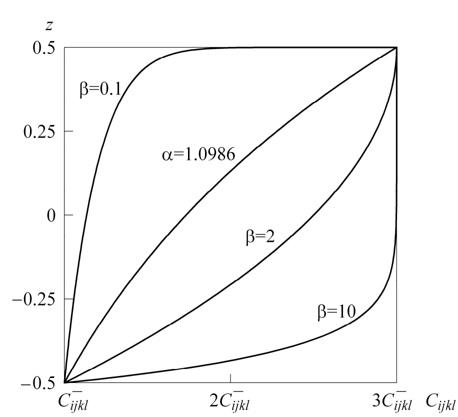

Fig. 2. Distribution of elastic constants through the thickness of the FG shell

$$\overline{D}_3 = 10^{12} D_3(L/2, 0, z); \quad z = \theta_3 / h.$$

Tables 1 and 2 demonstrate the high potential of the SaS method developed that yields the exact solution of 3D electroelasticity for FG piezoelectric cylindrical shells with a prescribed accuracy for the case of $\beta = 2$ using the sufficient number of SaS. Figs. 3 and 4 present the distributions of transverse stresses and electric displacement through the thickness of the shell with $\beta = 2$ employing nine SaS. As can be seen, the boundary conditions on the bottom and top surfaces for transverse stresses and electric displacement are satisfied with a high accuracy.

Table 1

Results for a FG piezoelectric cylindrical shell under mechanical loading

N	$\overline{u}_3(0)$	$\overline{\varphi}(0)$	$\overline{\sigma}_{11}(0{,}5)$	$\overline{\sigma}_{12}(0{,}5)$	$\overline{\sigma}_{13}(0)$	$\overline{\sigma}_{23}(0)$	$\overline{\sigma}_{33}(0)$	$\overline{D}_3(0)$
5	34,656	0,98923	24,414	–16,224	7,9632	–2,1846	0,39330	–6,5279
7	34,656	0,98922	24,413	–16,224	7,8825	–2,1634	0,39233	–6,4607
9	34,656	0,98922	24,413	–16,224	7,8944	–2,1658	0,39254	–6,4705
11	34,656	0,98922	24,413	–16,224	7,8922	–2,1656	0,39250	–6,4690
13	34,656	0,98922	24,413	–16,224	7,8926	–2,1655	0,39251	–6,4692

Table 2

Results for a FG piezoelectric cylindrical shell under electric loading

N	$\overline{u}_3(0)$	$\overline{\varphi}(0)$	$\overline{\sigma}_{11}(0{,}5)$	$\overline{\sigma}_{12}(0{,}5)$	$\overline{\sigma}_{13}(0)$	$\overline{\sigma}_{23}(0)$	$\overline{\sigma}_{33}(0)$	$\overline{D}_3(0)$
5	–3,4244	–1439,9	–5,2723	–4,4782	–18,852	23,062	0,15825	4153,7
7	–3,4244	–1439,9	–5,2917	–4,4781	–18,681	22,844	0,15632	4141,5
9	–3,4244	–1439,9	–5,2973	–4,4781	–18,705	22,875	0,15655	4144,0
11	–3,4244	–1439,9	–5,2984	–4,4781	–18,701	22,870	0,15652	4143,6
13	–3,4244	–1439,9	–5,2986	–4,4781	–18,702	22,871	0,15652	4143,6

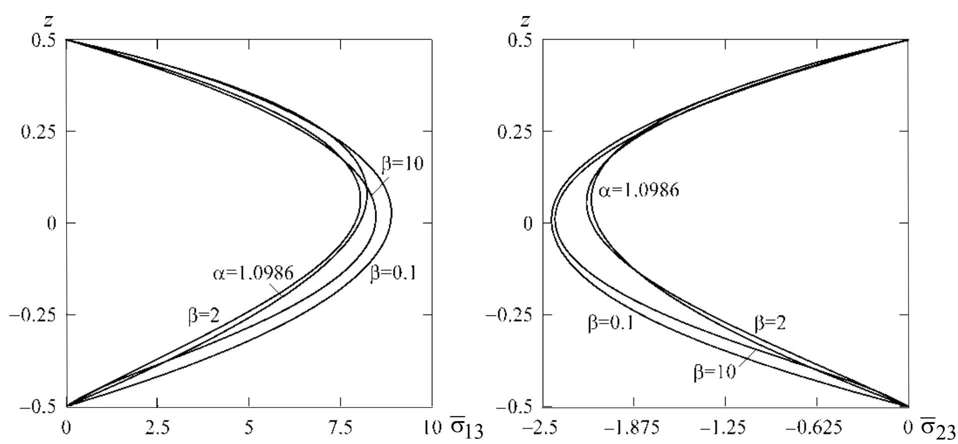

Fig. 3. Distributions of transverse stresses and electric displacement through the thickness of the FG piezoelectric cylindrical shell under mechanical loading (start)

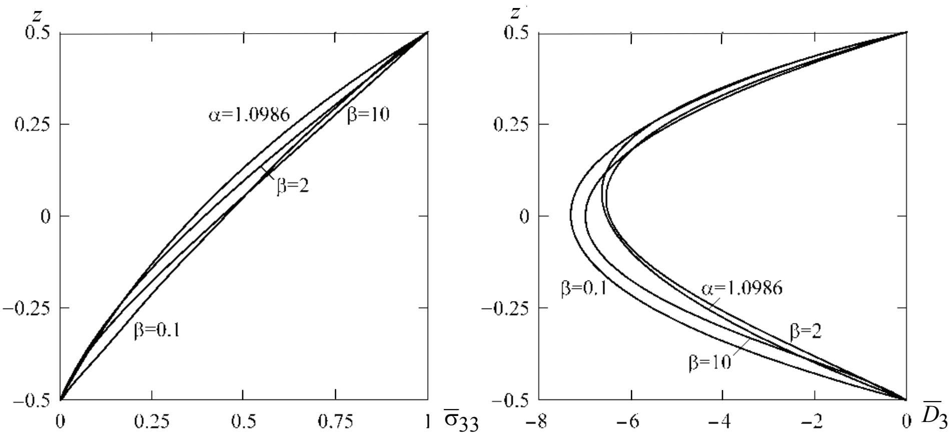

Fig. 3. Continued

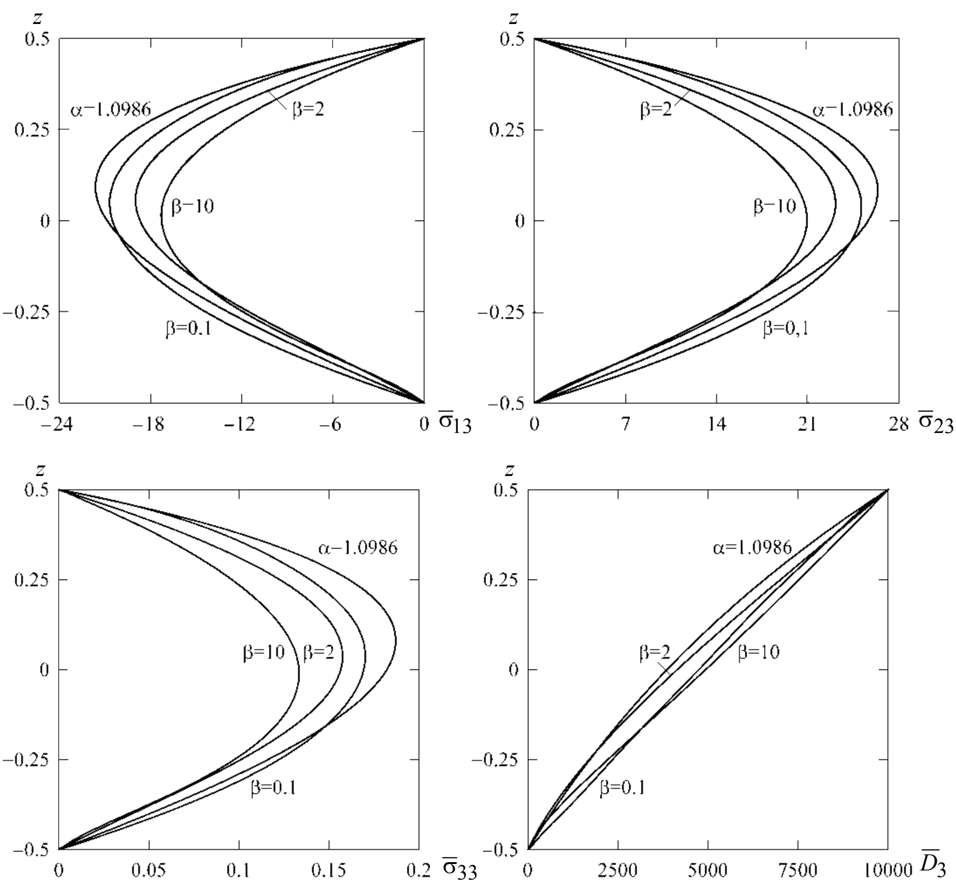

Fig. 4. Distributions of transverse stresses and electric displacement through the thickness of the FG piezoelectric cylindrical shell under electric loading

6. Conclusion

An efficient approach to 3D exact solutions of electroelasticity for FG piezoelectric cylindrical shells has been proposed. It is based on the new method of SaS located at Chebyshev polynomial nodes inside the shell body. The stress analysis is based on the 3D constitutive equations of electroelasticity and gives an opportunity to obtain the 3D exact solutions for thick FG piezoelectric cylindrical shells with a specified accuracy by using the sufficient number of SaS.

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Точное решение для цилиндрических оболочек из функциональных пьезоэлектрических материалов

Г.М. Куликов[1], А.В. Ерофеев[2]

Кафедры: «Прикладная математика и механика» (1), «Конструкции зданий и сооружений» (2), ФГБОУ ВПО «ТГТУ»; kulikov@apmath.tstu.ru

Ключевые слова и фразы: функциональный пьезоэлектрический материал; цилиндрическая оболочка; электроупругость; метод отсчетных поверхностей.

Аннотация: Представлен метод отсчетных поверхностей с приложением к цилиндрическим оболочкам из функциональных пьезоэлектрических материалов. Показано, что метод отсчетных поверхностей может быть эффективно использован для получения точных трехмерных решений для цилиндрических оболочек из функциональных пьезоэлектрических материалов с заданной точностью, используя достаточно большое число отсчетных поверхностей, размещенных в узловых точках полинома Чебышёва.

Genaue Lösung für die zylindrischen Hüllen aus den funktionalen piezoelektrischen Materialien

Zusammenfassung: Es ist die Methode der Ableseoberflächen mit der Anwendung zu den zylindrischen Hüllen aus denfunktionalen piezoelektrischen Materialien dargelegt. Es ist vorgeführt, dass die Methode der Ableseoberflächen für das Erhalten der genauen dreidimensionalen Lösungen für die zylindrischen Hüllen aus den funktionalen piezoelektrischen Materialien mit der aufgegebenen Genauigkeit wirksam sein kann, die genug große Zahl der Ableseoberflächen verwendend, die in den Knotenpunkten des Polynoms von Chebyshev aufgestellt sind.

Solution précise pour les enveloppes cylindriques à partir des matériaux fonctionnels piézo-électriques

Résumé: Est proposée la méthode des surfaces de repère avec une application pour les enveloppes cylindriques à partir des matériaux fonctionnels piézo-électriques. Est montré que la méthode des surfaces de repère peur être utilisée effectivement pour l'obtention des solutions précises de trois dimensions pour les enveloppes cylindriques à partir des matériaux fonctionnels piézo-électriques avec une précision donnée en utilisant une grande quantité de surfaces de repère situées dans les points-noeuds du polynôme de Tschebichev.

Авторы: ***Куликов Геннадий Михайлович*** – доктор физико-математических наук, профессор, заведующий кафедрой «Прикладная математика и механика»; ***Ерофеев Александр Владимирович*** – аспирант кафедры «Конструкции зданий и сооружений», ФГБОУ ВПО «ТГТУ».

Рецензент: ***Ярцев Виктор Петрович*** – доктор технических наук, профессор, заведующий кафедрой «Конструкции зданий и сооружений», ФГБОУ ВПО «ТГТУ».
