УДК 517.518

SUMMATION OF POWER SERIES OF FUNCTIONS OF CLASSES H_v^p ON BOUNDARY OF THE CONVERGENCE CIRCLE

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Key words and phrases: exponential summarizing sequences; weighted Hardy spaces; weighted norm estimates.

Abstract: The estimates of H_v^p-norm of maximal operators, generated by methods $\lambda_k(h) = \exp(-hu^{\alpha}(|k|)$, $k = 0, \pm 1, ...,$ $\alpha > 0$ of summation of power series $\varphi(\exp(ix)) \sim \sum_{k=0}^{\infty} \mu_k(\varphi)\exp(ikx)$ are obtained. The results are based on the estimates of L_v^p-norms of means of series and conjugated Fourier series of function $f(x) = \operatorname{Re}\varphi(\exp(ix))$.

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1. Hardy classes. A_p is condition. Let H_v^p be weighted Hardy space of all functions $\varphi = \varphi(z)$ of complex variable $z = r\exp(ix)$, $0 < r < 1$, $x \in Q$, which are analytic in a circle of $|z| < 1$, for which

$$\|\varphi\|_{v,p} = \sup_{0 \le r < 1} \int_Q |\varphi(r\exp(ix))|^p \, v(x)\, dx < \infty \text{ and } \operatorname{Im}\varphi(0) = 0. \quad (1)$$

Here, $v = v(x) \ge 0$ is fixed function from the class of measurable on $Q = (-\pi, \pi]$ and 2π-periodic functions.

It is said that any function f from this class belongs to weight space $L_v^p = L_v^p(Q)$, if

$$\|f\|_{v,p} = \left(\int_Q |f(x)|^p \, v(x) dx\right)^{1/p} < \infty, \quad p \ge 1.$$

In the case of Lebesque spaces $L^p = L^p(Q)$ we have for $v \equiv 1$; in particular, $L = L^1(Q)$. It is denoted as follows:

$$A_p(v;\Omega)=\left(\frac{1}{|\Omega|}\int_\Omega v(t)dt\right)\left(\frac{1}{|\Omega|}\int_\Omega v^{-1/(p-1)}(t)dt\right)^{p-1},\quad p\ge 1,$$

where Ω is arbitrary interval, and multiplier $\left(\int_\Omega v^{-1/(p-1)}(t)dt\right)^{p-1}$ is equal $\operatorname{esssup}_{t\in\Omega}\frac{1}{v(t)}$ for $p=1$ by definition.

It is said that A_p -condition of Muckenhoupt-Rozenblum [1, 2] is satisfied and the notation $v\in A_p$ is applicable, if $\sup_\Omega A_p(v;\Omega)<\infty,\ p\ge 1.$ In the present work, as well as in [1 – 3], we suppose $0\cdot\infty=0.$ Then

$$\left(\int_Q v^{-1/(p-1)}(t)dt\right)^{p-1}<\infty \text{ for } v\in A_p,\ p\ge 1,$$

since otherwise $\int_Q v(t)dt=0,$ but this trivial case of $v(t)\sim 0$ ($v(x)=0$ almost everywhere), we exclude from consideration.

It is possible to consider now, that every $\varphi\in H_v^p$ is a function from Hardy class H [4, vol. 1, p. 431], which corresponds to a case of $v\equiv 1,\ p=1.$ In fact

$$\int_Q|\varphi(t)|\,dt=\int_Q|\varphi(t)|\,v^{1/p}(t)v^{-1/p}(t)dt\le\left(\int_Q|\varphi(t)|^p v(t)dt\right)^{1/p}\left(\int_Q v^{-1/(p-1)}(t)dt\right)^{(p-1)/p}<\infty;$$

we have used the Helder inequality here for $p>1$ and the agreement on $\left(\int_\Omega v^{-1/(p-1)}(t)dt\right)^{p-1}$ for $p=1.$ It can be assumed (in just the same way), that every $f\in L_v^p(Q)$ is a function from the class $L(Q).$

We exclude a trivial case of $v(x)\sim\infty$ from consideration. Then $\int_Q v(x)dx<\infty,$ since otherwise A_p – a condition that implies the relation $\left(\int_Q v^{-1/(p-1)}(t)dt\right)^{p-1}=0,$ so that $v(x)\sim\infty.$ Let E be a set which is measurable by Lebesque. We introduce now the following measure of E: $\mu\{E\}=\int_E v(x)dx.$

In this paper we consider the so-called exponential means of expansions of analytical functions $\varphi\in H_v^p$ on the boundary of the convergence circle. In paragraph 3 we assert their relations with the corresponding means of Fourier series and conjugate Fourier series of functions $f(x)=\operatorname{Re}\varphi(\exp ix).$ In turn, the latest estimates are based on the properties of the maximal operators

$$f^*=f^*(x)=\sup_{\eta>0}\frac{1}{\eta}\int_{x-\eta}^{x+\eta}|f(t)|\,dt, \tag{2}$$

$$\tilde{f}^*=\tilde{f}^*(x)=\sup_{\eta>0}\left|\int_{\eta\le|t|\le\pi}\frac{f(x+t)}{2tg\,\frac{t}{2}}dt\right|. \tag{3}$$

The operators (2), (3) are defined for every $f \in L$ [4, vol. 1, p. 60–61, 401, 442, 443]; besides the conjugate function $\tilde{f}(x) = -\frac{1}{2} \lim_{\eta \to +0} \int_{\eta \le |t| \le \pi} f(x+t) \operatorname{ctg} \frac{t}{2} dt$ exists almost everywhere.

In papers [1, 3] the following results are shown:

– the boundedness of operators (2) and (3) from L_v^p in L_v^p is equivalent to condition $v \in A_p$ for every $p > 1$;

– the estimates of "week type"

$$\mu\{x \in Q \mid f^*(x) > \varsigma > 0\} \le C\left(\frac{\| f \|_{v,p}}{\varsigma}\right)^p, \quad \mu\left\{x \in Q \mid \tilde{f}^*(x) > \varsigma > 0\right\} \le C\left(\frac{\| f \|_{v,p}}{\varsigma}\right)^p \tag{4}$$

are equivalent to condition $v \in A_p$ for every $p \ge 1$.

Here, $C = C_{v,p}$ will represent a constant, though not necessarily one such constant.

2. Exponential methods of summation. Let $f \in L$, and

$$c_k(f) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \exp(-ikt)\, dt, \quad k \in Z \tag{5}$$

be a sequence of its complex Fourier coefficients. For this function we consider Fourier series

$$s[f, x] = \sum_{k=-\infty}^{\infty} c_k(f) \exp(ikx) \tag{6}$$

and conjugate Fourier series

$$\tilde{s}[f, x] = -i \sum_{k=-\infty}^{\infty} (\operatorname{sgn} k) c_k(f) \exp(ikx). \tag{7}$$

In various questions of the analysis there is a problem of behavior of families means of (5), (6)

$$U_h(f) = U(f, x; \lambda, h) = \sum_{k=-\infty}^{\infty} \lambda_{|k|}(h) c_k(f) \exp(ikx) \tag{8}$$

and

$$\tilde{U}_h(f) = \tilde{U}(f, x; \lambda, h) = -i \sum_{k=-\infty}^{\infty} (\operatorname{sgn} k) \lambda_{|k|}(h)\, c_k(f) \exp(ikx), \tag{9}$$

at $h \to +0$. Here,

$$\Lambda = \{\lambda_k(h),\ k = 0,1,...\} \tag{10}$$

is the arbitrary sequence infinite, generally speaking, determined by values of parameter $h > 0$. In a case of the discrete parameter h, a summability of Fourier series in points of Lebesgue and uniformly with respect to x on an interval of a continuity of function was studied by many authors [5]. In the general case, we say that the sequence (10) defines a semi-continuous method of summability; the most interest is represented by the regular methods of summability. Namely, we say that the method (10) is regular, if the convergence of the series (6) to $f = f(x)$ (in the point x or in the corresponding metric space) implies the convergence of means (8) to $f = f(x)$ at $h \to +0$. As it is shown in [6, p. 79], the regularity conditions of methods (10) are as follows:

$$\lambda_0(h) = 1,\ \lim_{h\to 0} \lambda_k(h) = 1,\ k = 0, 1, ..., \tag{11}$$

$$\sup_{h>0} \sum_{k=0}^{\infty} |\, \Delta\, \lambda_k(h) \,| < \infty. \tag{12}$$

In this paper we consider mainly the so-called exponential summation methods. Namely, we assume that

$$\lambda_0(h) = 1,\ \lambda_k(h) = \lambda(x,h)\,|_{x=k}\,,\ k = 1, 2, ...,\ \text{where } \lambda(x,h) = \exp(-hu^{\alpha}(x)),\ \alpha > 0, \tag{13}$$

and a non-negative function $u(x)$ is continuous on $[0,+\infty)$ and twice differentiable on $(0,+\infty)$. Specifically, when $h = \ln\frac{1}{r},\ 0 < r < 1,\ \tau(x) = x$ we have in (8), (9) a family of classical means (conjugated means) of Poisson – Abel

$$\sigma_r(f,x) = \sum_{k=-\infty}^{\infty} r^{|k|} c_k(f) \exp(ikx) \ \text{and}\ \tilde{\sigma}_r(f,x) = -i \sum_{k=-\infty}^{\infty} (\operatorname{sgn} k) r^{|k|} c_k(f) \exp(ikx). \tag{14}$$

3. The means of power series and Fourier series (conjugate series). Let's consider now $\varphi \in H$. The behavior of

$$\varphi(r\exp(ix)) = \sum_{k=0}^{\infty} \mu_k(\varphi) r^k \exp(ikx), \quad 0 < r < 1,\ x \in Q, \tag{15}$$

on the boundary of the convergence circle $(r \to 1)$, has been well studied. So [9, p. 541],

$$\varphi(\exp(ix)) = \lim_{r\to 1} \varphi(r\exp(ix)) = f(x) + ig(x) \tag{16}$$

exists almost everywhere. Here, $f, g \in L,$ and the coefficients $\mu_k(\varphi)$ in the expansion (15) can be estimated as

$$\mu_k(\varphi) = \frac{1}{2\pi} \int_Q \varphi(\exp(it)) \exp(-ikt)\, dt,\ \ k = 0, 1, ...; \tag{17}$$

it is natural to assume that $\mu_k(\varphi) = 0$ when $k < 0.$ If we put

$$\varphi(\exp(ix)) \sim \sum_{k=0}^{\infty} \mu_k(\varphi) \exp(ikx), \tag{18}$$

then (15) can be considered as a family of Poisson – Abel means of series (18) on the boundary of the convergence circle. Then it will be natural to consider a more general exponential means

$$\Theta_h(\varphi) = \Theta(\varphi, x; \lambda, h) = \sum_{k=0}^{\infty} \mu_k(\varphi) \lambda_k(h) \exp(ikx) \tag{19}$$

of the series (18), where $\lambda_k(h)$ are defined in the form of (13). The following statement establishes a relation between the families (19), (8), (9).

Theorem 3.1. If $\varphi \in H$ and $f(x) = \operatorname{Re}\varphi(\exp(ix)),$ then the representation

$$\Theta(\varphi, x; \lambda, h) = U(f, x; \lambda, h) + i\,\tilde{U}(f, x; \lambda, h) \tag{20}$$

holds. In particular (see (14)), $\varphi(r\exp(ix)) = \sigma_r(f,x) + i\,\tilde{\sigma}_r(f,x).$

The proof of (20) will be based on the arguments similar to [7, p. 542 – 545]. Firstly, we prove that the coefficients of (17) are related to the Fourier coefficients (5) of function $f = f(x) = \operatorname{Re}\varphi(\exp(ix))$ as follows:

$$\mu_0(f) = c_0(f),\ \ \mu_k(f) = 2c_k(f),\ \ k = 1, 2, ... \tag{21}$$

We have

$$\mu_k(f) = c_k(f) + ic_k(g),\ \ k = 0, 1, 2, ..., \tag{22}$$

so that

$$c_0(g) = \operatorname{Im}\mu_0(\varphi) = \operatorname{Im}\varphi(0) = 0. \tag{23}$$

Further,

$$ic_k(g) = (\operatorname{sgn} k)c_k(f). \tag{24}$$

Indeed, for $k < 0$ the equality (24) is equivalent to $c_k(f) + ic_k(g) = \mu_k(\varphi) = 0,$ and for $k > 0$ it follows from the relation

$$\int_Q (f(x) - ig(x))\exp(-ikx)\,dx = 0,$$

which holds as its real and imaginary parts are equal, respectively, to the real and imaginary part of the obvious equality

$$\int_Q (f(x) + ig(x))\exp(ikx)\,dx = \mu_{-k}(\varphi) = 0.$$

Thus, we see that (21) there is a consequence of (22) – (24).

It should be noted that, according to (21), the right-hand side of (20) takes the form

$$c_0(f) + 2\sum_{k=1}^{\infty}\lambda_k(h)c_k(f)\exp(ikx) = \sum_{k=0}^{\infty}\lambda_k(h)\mu_k(\varphi)\exp(ikx),$$

and this is the assertion of Theorem 3.1.

Now the study of means (19) reduces to the study of means (8) and (9).

4. The estimates of maximal operators generated by exponential summation methods. Let's refer to the case of (13). The means (8), (9) and (19) are re-denoted through

$$U_h(f) = U(f, x; u^\alpha, h),\ \ \tilde{U}_h(f) = \tilde{U}(f, x; u^\alpha, h) \text{ and } \Theta_h(\varphi) = \Theta(\varphi, x; u^\alpha, h)$$

respectively. Let

$$\Theta_*(\varphi) = \Theta_*(\varphi, x; u^\alpha) = \sup_{h>0} |\Theta(\varphi, x; u^\alpha, h)|;$$

$$U_*(f) = U_*(f, x; u^\alpha) = \sup_{h>0} |U(f, x; u^\alpha, h)|;\ \ \tilde{U}_*(f) = \tilde{U}_*(f, x; \lambda) = \sup_{h>0} |\tilde{U}(f, x; u^\alpha, h)|$$

Theorem 4.1. Suppose (see (13)) $u''(x) < 0$ on $(0, +\infty)$, $0 < \alpha \le 1$, and

$$\exp(-hu^\alpha(x))\ln x = O(1),\ \ x \to +\infty. \tag{25}$$

for every $h > 0$. If $v \in A_p$, then the estimates

$$\| \Theta_*(\varphi) \|_{v,p} \le C \| \varphi \|_{v,p},\ \ p > 1; \tag{26}$$

$$\mu\{x \in Q \mid \Theta_*(\varphi, x; u^\alpha) > \varsigma > 0\} \le C\left(\frac{\| \varphi \|_{v,p}}{\varsigma}\right)^p,\ \ p \ge 1 \tag{27}$$

hold. The estimates remain valid for every $\alpha > 0$ under the condition that a function

$$V = V(x) = \alpha h u^{\alpha} (u')^2 - (\alpha - 1)(u')^2 - u u'', \quad \alpha > 0$$

has a finite number of zeros, the condition (25) holds and there is a constant $C = C_{u,\alpha}$, such that

$$x h \exp(-h u^{\alpha}(x)) u^{\alpha-1}(x) | u'(x) | \le C_{u,\alpha}, \tag{28}$$

for all $h > 0$, $x \in (1, +\infty)$.

As it follows from (20), the estimate (26) will be established if we prove that under the conditions of Theorem 4.1, the inequality

$$\| U_*(f) \|_{v,p} + \| \tilde{U}_*(f) \|_{v,p} \le C \ \| (f) \|_{v,p}, \quad p > 1 \tag{29}$$

holds and note that $| f(x) | \le | \varphi(\exp(ix)) |$. Next, to prove (27) it will be sufficient to establish that

$$\mu\{x \in Q \mid U_*(f, x; \lambda) > \varsigma > 0\} \le C \left(\frac{\| f \|_{v,p}}{\varsigma} \right)^p, \quad p \ge 1 \tag{30}$$

and

$$\mu\{x \in Q \mid \tilde{U}_*(f, x; \lambda) > \varsigma > 0\} \le C \left(\frac{\| f \|_{v,p}}{\varsigma} \right)^p, \quad p \ge 1, \tag{31}$$

because, according to (20),

$$\{x \in Q \mid \Theta_*(\varphi, x; \lambda) > \varsigma > 0\} \subset \left\{ x \in Q \,|\, U_*(\varphi, x; \lambda) > \frac{\varsigma}{2} > 0 \right\} \cup \left\{ x \in Q \mid \tilde{U}_*(\varphi, x; \lambda) > \frac{\varsigma}{2} > 0 \right\}.$$

In turn, the estimates (29) – (31) will follow from the results of [1, 3], cited in paragraph 1 (in particular, see (4)), if we prove that

$$U_*(f, x; \lambda) \le C f^*(x) \ \text{ and } \ \tilde{U}_*(f, x; \lambda) \le C \left(f^*(x) + \tilde{f}^*(x) \right) \tag{32}$$

for almost all x.

5. Auxiliary statements. Sequence (10) is convex (concave) if $\Delta_k^2 = \Delta^2 \lambda_k(h) > 0 \ (\Delta_k^2 < 0)$, where

$$\Delta_k^2 = \Delta_k - \Delta_{k+1}, \ \Delta_k = \Delta\lambda_k = \lambda_k(h) - \lambda_{k+1}(h), \ k = 0, 1, \ldots$$

Sequence (10) is piecewise convex if Δ_k^2 changes sign a finite number of times, $k = 0, 1, \ldots$ In [7] established in the following assertion.

Lemma 5.1. If the sequence (3) is convex (concave) and the relation

$$\lambda_N(h) = O\left(\frac{1}{\ln N} \right), \ N \to \infty, \tag{33}$$

is valid for every $h > 0$, then the estimates

$$U_*(f, x; \lambda) \le C f^*(x) \sum_{k=0}^{\infty} (k+1) | \Delta^2 \lambda_k(h) |, \tag{34}$$

$$\tilde{U}_*(f, x; \lambda) \le C \left(f^*(x) + \tilde{f}^*(x) \right) \sum_{k=0}^{\infty} (k+1) \left| \Delta^2 \lambda_k(h) \right|. \tag{35}$$

hold almost everywhere. The estimates remain valid for piecewise convex sequences (10) if (33) holds and there is a constant $C = C_\Lambda$, such that

$$|\lambda_k(h)| + k\,|\Delta\lambda_k(h)| \le C_\Lambda \tag{36}$$

for every $h > 0$, $k = 1, 2, ...$

To establish (32), it is now sufficient to observe that

1) for every sequence (10), which is convex (concave) or piecewise-convex and satisfies (27), we have (see [8])

$$\sum_{k=0}^{\infty} (k+1)\,|\Delta^2\lambda_k(h) < C \text{ with a constant } C = C_\Lambda; \tag{37}$$

2) the following auxiliary assertion occurs

Lemma 5.2. Under the conditions of Theorem 4.1, the sequence (13) is convex and satisfies (33) with $0 < \alpha \le 1$; (13) is piecewise convex and satisfies (33) and (36) with $\alpha > 1$. In both cases, the summation method (13) is regular.

The first assertion is a consequence of the Abel transform [4, vol. 1, p. 15], and conditions (25), (28). Regularity condition (11) follows from (13) in an obvious way; the condition (12) follows from

$$\sum_{k=0}^{N} |\Delta\,\lambda_k(h)| = \sum_{k=0}^{N} ((k+1)-k)\,|\sum_{\mathrm{j=k}}^{\infty} \Delta^2\lambda_j(h)| =$$

$$= N\,|\Delta\lambda_N| + \sum_{k=0}^{N-1} (k+1)\left(|\sum_{\mathrm{j=k}}^{\infty} \Delta^2\lambda_j(h)| - |\sum_{\mathrm{j=k+1}}^{\infty} \Delta^2\lambda_j(h)|\right) \le C\left(1 + \sum_{k=0}^{\infty} (k+1)\,|\Delta^2\lambda_k(h)|\right). \tag{38}$$

Upon receipt of the estimate (38) it was used the Abel transform and uniform (in *N*) boundedness of productions of the type $N\,|\Delta\lambda_N|$, see [4, vol. 1, p. 156]. Now (32) is installed and Theorem 4.1 is completely proved.

6. Results of convergence.

Theorem 6.1. Suppose that $v \in A_p$ and the conditions of Theorem 4.1 for sequence (13) are valid (corresponding to the cases $0 < \alpha \le 1$ and $\alpha > 1$). Then the relation

$$\lim_{h \to 0} \Theta_h(\varphi) = \varphi$$

holds μ-almost everywhere for each $f \in H_v^p$, $p \ge 1$ and in metric H_v^p for any $p > 1$.

According to (20) it is sufficient to prove that the relations

$$\lim_{h \to 0} U_h(f) = f, \tag{39}$$

$$\lim_{h \to 0} \tilde{U}_h(f) = \tilde{f} \tag{40}$$

hold μ-almost everywhere for each $f \in L_v^p$, $p \ge 1$ and in metrics L_v^p for any $p > 1$. In turn, assertions (39) and (40) in a standard way (see [4, vol. 2, p. 464–465]) follow from (29) – (31) and (11).

7. Examples. It is easy to verify that the conditions of Theorem 4.1 are satisfied in the following cases.

1) $u(x) = \ln x$, so that

$$\lambda_0(h) = 1,\ \lambda(x,h) = \exp\left(-h \ln^\alpha x\right),\ x > 0,\ \alpha > 0.$$

2) $u(x) = x$, so that

$$\lambda_0(h) = 1,\ \lambda(x,h) = \exp\left(-hx^{\alpha}\right),\ x > 0,\ \alpha > 0.$$

References

1. Muckenhoupt B. *Trans. Amer. Math. Soc.*, 1972, vol. 165, pp. 207-226.
2. Rozenblum M. *Trans. Amer. Math. Soc.*, 1962, vol. 105, pp. 32-42.
3. Hunt R., Muckenhoupt B., Wheeden R. *Trans. Amer. Math. Soc.*, 1973, vol. 176, pp. 227-251.
4. Zygmund A. *Trigonometricheskie ryady* (Trigonometric Series), Cambrifge, 1959.
5. Nikol'skii S.M. *Mathematics of the USSR - Izvestiya*, 1948, no. 12, pp. 259-278.
6. Cooke R.G. *Beskonechnye matritsy i prostranstva posledovatel'nostei* (Infinite matrices and sequence spaces), Moscow: Gosudarstvennoe izdatel'stvo fiziko-matematicheskoi literatury, 1960, 471 p.
7. Nakhman A.D., Osilenker B.P. *Transactions of the Tambov State Technical University*, 2014, vol. 20, no. 1, pp. 101-109.

Суммирование степенных рядов функций классов H_v^p на границе круга сходимости

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Ключевые слова и фразы: весовые пространства Харди; оценки весовых норм; экспоненциальные суммирующие последовательности.

Аннотация: Получены оценки H_v^p -норм максимальных операторов, порожденных экспоненциальными методами суммирования степенных рядов $\varphi(\exp(ix)) \sim \sum_{k=0}^{\infty} \mu_k(\varphi)\exp(ikx)$. Результаты основаны на оценках L_v^p -норм средних рядов и сопряженных рядов Фурье функции $f(x) = \mathrm{Re}\,\varphi(\exp(ix))$.

Список литературы

1. Muckenhoupt, B. Weighted Norm Inequalities for the Hardy Maximal Function / B. Muckenhoupt // Trans. Amer. Math. Soc. – 1972. – Vol. 165. – P. 207 – 226.
2. Rozenblum, M. Summability of Fourier Series in $L^p(d\mu)$ / M. Rozenblum // Trans. Amer. Math. Soc. – 1962. – Vol. 105. – P. 32 – 42.
3. Hunt, R. Weighted Norm Inequalities for Conjugate Function and Hilbert Transform / R. Hunt, B. Muckenhoupt, R. Wheeden // Trans. Amer. Math. Soc. – 1973. – Vol. 176. – P. 227 – 251.
4. Зигмунд, А. Тригонометрические ряды : пер. с англ. : в 2 т. / А. Зигмунд. – М. : Мир, 1965. – 2 т.

5. Никольский, С. М. О линейных методах суммирования рядов Фурье / С. М. Никольский // Известия АН СССР. Отделение мат. и естеств. наук. Сер. мат. – 1948. – № 12. – С. 259 – 278.

6. Кук, Р. Бесконечные матрицы и пространства последовательностей : монография / Р. Кук. – М. : Гос. изд-во физ.-мат. лит., 1960. – 471 с.

7. Nakhman, A. D. Exponential Methods of Summation of the Fourier Series / A. D. Nakhman, B. P. Osilenker // Вестн. Тамб. гос. техн. ун-та. – 2014. – Т. 20, № 1. – С. 101 – 109.

Summierung der Kraftreihen der Funktionen der Klassen H_ν^p an der Grenze des Kreises der Konvergenz

Zusammenfassung: Es sind die Einschätzungen der H_ν^p Normen der maximalen Operatoren, die von den experimantalen Methoden der Summierung der gesetzten Reihen $\varphi(\exp(ix)) \sim \sum_{k=0}^{\infty} \mu_k(\varphi)\exp(ikx)$ angegen. Die Ergebnisse sind auf den Einschätzungen der L_ν^p-Normen der mittleren Reihen und der verknüpften Fourierreihen der Funktion gegründet $f(x) = \operatorname{Re}\varphi(\exp(ix))$.

Sommation des séries puissance des fonctions des classes H_ν^p sur la frontière du cercle de convergence

Résumé: Sont obtenues les estimations H_ν^p des normes maximales des opérateurs générées par les méthodes exponentielles de la sommation des séries puissance $\varphi(\exp(ix)) \sim \sum_{k=0}^{\infty} \mu_k(\varphi)\exp(ikx)$. Les résultats sont basés sur les estimations L_ν^p des normes des séries moyennes et des séries de configuration de Fourier de la foncion $f(x) = \operatorname{Re}\varphi(\exp(ix))$.

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