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# MATHEMATICAL TRAINING IN THE SYSTEM "SCHOOL – HIGHER EDUCATIONAL INSTITUTION" UNDER THE CONCEPTION OF DEVELOPMENT OF MATHEMATICAL EDUCATION IN RUSSIA

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**Key words and phrases:** contents of mathematical training; innovative component of target block; mathematical education system.

**Abstract:** Under the Conception of development of mathematical education the transformation of the purposes of mathematical training from traditional to innovative ones is considered. Questions of formation of the updated content of mathematical education in the system "school – higher educational institution" at a transitional stage "Secondary school – undergraduate elementary level" are discussed.

**Introduction.** The Conception of development of mathematical education in Russia was introduced by the Order of the Government of the Russian Federation on December 24, 2013, after one and half a year discussion [1]. Its foundation was formed by the conclusions made on the basis of analytical data on the condition of mathematical training at various educational levels. The purpose of actions for the Conception implementation in the system "school – higher educational institution" is expressed most capaciously by the thesis: "studying and teaching of mathematics, on the one hand, provides readiness of learners for mathematics application in other areas, and, on the other hand, has a backbone function, influencing significantly intellectual readiness of school and university students for training, and understanding the content of other subjects".

The main ideas of the Conception is to set a vector of development of mathematical education at all levels – from preschool to higher school and postgraduate education – for the next decades. Now, theoretical understanding,

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approbation, development and embodiment in concrete actions are necessary for these ideas.

In fact, the structure of the system of mathematical education under the Conception comprises the following components: the target block; the "integrated" elements (subsystems) and connections between subsystems [2]; the "output" of the system; the function of the system as a way of manifestation of its properties in this set of the relations with educational and cultural environment.

In the given work we offer the analysis of: 1) the target block of the concept; 2) the organizational block; 3) the block of the content of mathematical training. Besides, we mostly focus on those innovative approaches which, from our point of view, correspond to the main ideas of the Conception.

**1. Purposes of mathematical education: traditions and innovations.** As a conceptual basis of improvement of mathematical education we understand a combination of its best settled traditions and the approved innovations. Thus, the definition of innovations in education assumes [3] that these innovations:

**Purposes of mathematical education**

| Traditional purposes | Innovative component |
|---|---|
| – Intellectual development of learners, formation of qualities of thinking, which are characteristic of mathematical activity and necessary for a person for a full-fledged life in society | – Development of abilities of logical and critical thinking, designing, communication and interaction using wide mathematical material, overcoming intellectual obstacles |
| – Formation of understanding of ideas and methods of mathematics as a form of the description and as a method of perception of reality, and a wide range of its applications | – Strengthening of the backbone nature of mathematical education in high-quality training of a modern specialist; a practice-orientated teaching of mathematics, development of abilities of mathematical modeling (creation of models of reality and interpretation of results), mathematics application, including, the ICT use |
| – Mastering certain mathematical knowledge and skills which are necessary for application in practical activities, for studying of related subjects, for further education | – Updating of the content of mathematical education: it has to be replenished more and more with elements of applied and "computer" mathematics;<br>– Updating of the nature of mathematical activity: preserving its traditional character (problem-solving, proof of theorems), it has to occur in the ICT environment, with ICT tools application |
| – Development of personality in the course of studying mathematics and doing mathematical activity | – Development of personal qualities for social mobility, ability to make independent decisions;<br>– Teaching to overcome cogitative stereotypes, developing the abilities to express personal point of view and to reason it, to recognize one's mistakes and to correct them |

are caused by a certain social order and are directed to the improvement of development, training, education; are focused on practice; can be introduced and spread (diffused) on the basis of approbation results.

In the target component of the Conception, which can be considered as general for all the levels of mathematical education (from preschool to higher school and postgraduate), we observed the following transformations of the purposes from traditional to innovative ones, cf. table.

**2.** We shall discuss **the purposes in relation to a transitional stage "Higher school as an initial step of a Bachelor degree".**

Secondary school – undergraduate elementary level.

We can distinguish the following features of current state of mathematical training at this stage:

– inflow (first of all, in the economic and information directions) of a large number of applicants, poorly motivated for mathematical activity (including fee-paying students);

– falling of prestige of engineering education (where mathematical knowledge and skills are most demanded), and as a result of it, lack of serious competition in the corresponding areas of education;

– considerable reduction of classroom hours with transferring of emphasis on independent work that is problematic in the conditions of students' lack of sufficient motivation, a low level of development of all-educational abilities and the most important of them – the ability to study;

– introduction of the course of mathematics into General Educational Programmes (**GEP**) of humanitarian directions of preparation for a bachelor degree.

At the same time, in the chapter "Professional education" of the Conception it is said [1]: "The system of professional education has to provide the necessary level of mathematical training of specialists for the needs of mathematical science, economy, scientific and technical progress, safety and medicine".

We can see the ways of solving the arising contradiction, first of all, in clear division of the purposes (planned levels of mathematical preparation) according to the following differentiation of groups of pupils:

1) classes of high secondary school with profound mathematical training – the bachelor/master degree in the field of mechanical-mathematical sciences and information technologies. According to the point of view of the group of mathematicians of Moscow State University, the authors of one of the projects of the Conception [4], "deep and informal studying of certain sections of mathematics and applied sciences close to it" is seen as bases for further scientific researches and practical activities;

2) profile classes – natural-science, technical, economic profiles of a bachelor degree. Here, first of all, it is necessary to develop general mathematical culture for the use of the gained knowledge and skills in further work in the chosen profession;

3) classes of a humanitarian profile (basic mathematical training) – the humanitarian directions of a bachelor degree: formation of logical culture, ability to analyze, classify, make hypotheses, to disprove them or to prove their solvency, to use analogies, to reason arguments etc.

However, the achievement of the set goals is impossible without strengthening motivation for mathematical activities in all groups of learners, bigger availability and openness of mathematical education, strengthening of ties within the line "school – higher educational institution". And here the components of the organizational block come to the forefront.

**3.** We shall list **actions for improvement of mathematical education in the system "school – higher educational institution"** (we shall limit the list of actions to the level of the region). In our opinion, they can be the following:

– providing all school students with a free access to electronic educational and scientific resources of libraries of higher educational institutions;

– organization on the basis of higher education institutions of mathematical Olympiads for school students with granting application privileges in this higher educational institution to winners;

– development of adaptive ("transitional") courses and corresponding training materials (manuals, workbooks, etc.) for students with poor school mathematical preparation;

– completion of development of GEP in mathematics, realizing competence-based approach in higher professional education (**HPE**);

– development and delivering mathematical elective courses for the most motivated students;

– involvement of students in the interdisciplinary scientific researches using mathematical modeling;

– formation of common educational and scientific information environment of higher educational institution promoting partial computerization of mathematical education in the higher educational institution;

– performing actions (at regional level) to monitor the process of formation and development of mathematical competence of students of nonmathematical directions for the purpose of comparison of the results of higher education institutions, tracking of dynamics of the process, exchange and distribution of the best practices of preparation.

**4. Content of training.** In all the projects of the Conception the question of the content of mathematical education both in school and high school courses was given a lot of attention. We share the point of view of the authors [4] that the most important result of training is *knowledge*. It has to be given "in the systematized state, taking into account interrelations between various mathematical and natural-science disciplines. One of the most important points is to fix the list of the basic mathematical concepts and facts on the government level", subject to studying in each direction of a bachelor degree.

At the same time we offer a universal core of the content of mathematical training, including both traditional and innovative (meeting modern requirements of science and practice) lines. The idea of their realization as means of mathematical modeling is innovative. The basic concepts and facts which are the center of the contents are invariant (in relation to the directions of professional activity) whereas their volume and extent of justification are variable. It is a question of the following substantial lines.

1. *Discrete mathematical line.*

   1.1. Sets, operations over sets. Cartesian products and relations over sets.

1.2. Finite sets. Finding the number of elements (principles of addition and multiplication). Combinatory formulas.
1.3. Sequences. Recursive definition. Recursions as a construction tool of algorithms.
1.4. Concept of Boolean algebras. Algebras of sets, events, propositional algebra [5].
1.5. Elements of the theory of graphs.

2. *Analytical-geometrical line.*
2.1. Rectangular and polar systems of coordinates as a means of definition of objects provisions on the plane.
2.2. Rectangular, cylindrical and spherical systems of coordinates as a means of definition of objects provisions in space.
2.3. Scalar and vector fields. Linear and nonlinear operations over vectors.
2.4. Equations of straight lines and planes and their relative positioning.

3. *Modeling of processes by methods of mathematical analysis.*
3.1. Function as dependence of an output parameter on input parameters of the process.
3.2. Limit behavior of function as a model of tendencies of the process.
3.3. Average and instant speeds of the process. Derivative. Differential (function "in the small" change) and its applications to approximate calculations.
3.4. Primitive as the restored processes. Introduction into the differential equations.
3.5. Process speeds in the set directions. Private derivatives and gradient.
3.6. General scientific concept of integration and integration in mathematics on the example of calculation of the area of a curvilinear trapezoid. Concept of a definite integral in general and its appendices.
3.7. Series as infinite sums. Application of methods of the mathematical analysis to research of convergence of numerical series.
3.8. Application of methods of mathematical analysis to computing tasks.

4. *Probabilistic and statistical line.*
4.1. Probability as a function on algebra of events. Classical, statistical and geometrical models. Probabilities of "compound" events.
4.2. Bernoulli's scheme. Binomial random variable. Concept of a random variable generally.
4.3. Distribution of discrete and continuous random variables. Function and density of distribution.
4.4. Numerical characteristics of distributions.
4.5. Classic distribution: uniform, Poisson, normal.
4.6. Theoretical and empirical distributions. Tabular and geometrical interpretations of sampling.
4.7. Numerical characteristics of sampling. Manifestation of the law of large numbers in the stability properties of the relative frequency

and consistency of the point estimates of the distribution parameters. Concept of interval estimates.

4.8. Statistical hypotheses of character and parameters of theoretical apportionment.

**5. Coordination of questions of the content of mathematical training.** One of the problematic zones of the transitional stage "Secondary school-an initial step of a Bachelor degree" is a coordination of questions and volume of the content of a mathematical material in "boundary" modules: stochastics, mathematical analysis and analytical geometry. Here, it is necessary to minimize such risks as duplications and formations of "lacunas", i.e. situations when certain questions are not taught in a school course, but they are basic for studying of subjects in a higher school course.

We shall formulate the following suggestions in relation to a basic level of development of mathematics in secondary school.

Introduction into analytical geometry, from our point of view, should be limited to the following issues.

1. Initial data on vectors: vector as a way of definition of movement on the plane and in space, linear operations over vectors (for example, addition of forces), and also linear operations in coordinates.

2. The equations of "horizontal", "vertical" and "inclined" straight lines on the plane"; parabola equations as graphics of square dependence and hyperbole as graphics of inversely proportional dependence; circle equation; condition of crossing of lines on the plane. At the same time we consider studying of the equation of the plane in space is impractical as within a school course a full "arsenal" of means of vector algebra for research of a reciprocal positioning of straight lines and the planes in space doesn't find a place.

3. Transformation of the equations of lines by parallel shift of system of coordinates on the corresponding vector is quite comprehensible on a basic level of studying mathematics and it clears up ways of creation of graphs of not only the above mentioned dependences, but also of transcendental functions of linear argument.

In our opinion, the basics of mathematical analysis in a school course have to include:

– concept of a limit, increment of function, derivative and antiderivative;

– geometrical sense of a derivative and tangent equation;

– derivative application to research of functions and finding of the greatest and smallest values of functions on a segment.

From our point of view, in concept of a limit it is enough to be limited to its following "dynamic" model: number $A$ is a function limit in point $a$, if (at movement of point $x$ on the abscissa axis to point a) the distance from $y = f(x)$ values to $A$ (on the ordinate axis) can be made, is set small if only the distance from $x$ to a is enough small. Accurate definition in the "epsilon-delta" language, as a rule, isn't perceived by the majority of pupils as it isn't supported with enough number of examples and isn't necessary in a further course.

We also consider impractical the study at a basic level of "the second remarkable limit", number "e" and natural logarithms because of impossibility of forming of the corresponding evidential base; the above mentioned concepts and facts which do not have further development and applications in a school

course are perceived by students as artificially introduced. Full consideration of all questions connected with integral calculus (except for the concept of an antiderivative and the simplest examples of its finding), in our opinion, has to be transferred to a university course.

In pedagogical circles the discussion about the expediency of studying of the beginnings of stochastics in a school course at a basic level proceeds. To put it more exactly, the main disputes consider the volume of studied concepts, the facts and the level of exactness of their reproduction. There are also extremely radical opinions. So, in the project of the Conception [4], the following is said: "it is necessary to shift for later time, and it is even better, in general to exclude from basic mathematical education all the material connected with probability theory and mathematical statistics, combination theory, the theory of sets and logic".

From our point of view, the probabilistic and statistical line in a school course can be presented only by elementary concepts and facts. Here more than in other sections of a course of mathematics the intuition and life experience of pupils find their application. Formalization of the basic facts has to give its way to their verbal characteristics (for example, addition of events corresponds to a phrase "at least one of the events" etc.), and standard probabilistic schemes to be offered in a simplified way (see, e.g., [5, 6]). Deeper studying can be organized only within a university course as it demands that students should have a sufficient level of algebraic knowledge (for example, understand the concept of normalized countable additive function in Boolean algebra) and knowledge in the field of the mathematical analysis.

Other practical measures for minimization of the above-noted risks within a transitional stage "secondary school – an initial step of a Bachelor degree" can be built, in our opinion, on the basis of activity of the working groups (consisting of mathematics teachers and high school teachers) in order to coordinate questions of the content of mathematical preparation.

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**Математическая подготовка в системе «школа – вуз» в условиях реализации концепции развития российского математического образования**

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**Ключевые слова и фразы:** инновационный компонент целевого блока; система математического образования; содержание математической подготовки.

**Аннотация:** В контексте Концепции развития математического образования рассмотрены трансформации целей математической подготовки от традиционных к инновационным. Обсуждаются вопросы формирования обновленного содержания математического образования в системе «школа – вуз» на переходном этапе «старшая школа – начальная ступень бакалавриата».

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