

## SOLVING THE PROBLEM OF AIRCRAFT BASIC PARAMETERS OPTIMIZATION BY NUMERICAL METHOD

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**Key words and phrases:** cruising speed; main parameters of aircraft; objective function; optimization by numerical methods; parametric analysis; prime cost of solution; takeoff mass.

**Abstract:** In this paper the authors analyzed optimization problems, using the example of transport aircraft shape designing, during the development of technical proposals. We used a numerical method for solving the problem of aircraft parameters optimization. Numerical solution method is based on the using of MathCAD system. Optimality criterion was used in the solution of the problem, namely, the criterion of “cost – effectiveness”. We considered the two vector of aircraft basic parameters optimization, on the one hand, its aerodynamic and energy parameters, and, on the other hand its parameters are set by the customer (company, organization that will be operating the aircraft).

Optimization problem is relevant to many industries and is widely reported in the scientific literature. In particular, problems of optimizing the main parameters of the aircraft are considered in [1]. When optimization problem is posed, there is a problem of choosing methods, which, probably, uniquely define the order of operations, that leading to a solution.

While solving the problem of aircraft basic parameters optimization, we can use absolute parameters, used in the design process, specifically: takeoff mass  $m_0$ , kg, wing area  $S$ ,  $m^2$ , and total starting thrust of engines  $P_0$ , N, or starting power  $N_0$ , kW.

In most cases, especially in the early stages of design, more convenient to use relatively main parameters of the aircraft: specific load on the wing area

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$p = mg/(10S)$ ,  $N/m^2$ , and thrust-to-weight  $P_t = 10P/(mg)$ , where  $m$  is takeoff mass;  $P$  is total thrust of aircraft engines.

These parameters depending from aircraft mass changing during flight, so while designing, are primarily determined their starting values (value in the beginning of flight, at the beginning of takeoff roll), i.e.  $p_0 = m_0g/(10S)$  and  $P_{t0} = 10P_0/(m_0g)$  [2].

Designing problem is formulated as follows: need to find the vector of parameters characterizing the shape, structure and size of the aircraft, which would meet requirements and restrictions, imposed on the projected airplane, and achievement of the minimum (maxima) of the objective function.

Note that while designing the takeoff mass plays a dual role. On the one hand, the mass is a dimensional parameter of aircraft, and process of selecting and harmonizing remaining parameters of the aircraft often begins from definition of mass. On the other hand, it is an essential generalized characteristic of aircraft, in terms of design results.

Analysis of relationships between parameters and flight characteristics of the aircraft shows:

1) values of flight characteristics depend only on its relative (specific) parameters  $p$ ,  $P_0$ , coefficient of aerodynamic lift  $c_y$ , drag coefficient  $c_x$ , aerodynamic qualities  $K$ , specific fuel consumption of the engine  $C_c$ , specific fuel mass  $m_f$ ;

2) regardless of the aircraft size to meet requirements of technical task for flight characteristics, aircraft must have a strictly defined set of needs relative parameters.

At the same time, parameters, such as  $p$ ,  $c_y$ ,  $c_x$ ,  $K$  in a generalized form defining external shape of the aircraft and its dimensions, while parameters  $P_0$ ,  $C_c$ ,  $m_f$  expressing specific amount of energy and specific traction forces, which engines (energy converters) must provide.

This determines the special significance of parameters  $p$  and  $P_0$ , which are sometimes called “core” parameters of the aircraft.

The considered aerodynamic and energetic parameters  $c_y$ ,  $c_x$ ,  $K$ ,  $m_f$ ,  $C_c$  at the same time are functions of geometric parameters of airframe and gas-dynamic parameters of engine, the most important of which are [2]:

– wing aspect ratio  $\lambda_w$ , empennage  $\lambda_e$ , fuselage  $\lambda_f$  ( $\lambda_{f.f}$  is elongation forward fuselage,  $\lambda_{a.f}$  – elongation aft fuselage);

– relative thickness of the wing  $c_w$  and relative thickness of the empennage  $c_e$ ;

– sweep angle of the wing  $\chi_w$  and sweep angle of the empennage  $\chi_e$ ;

– narrowing of the wing  $\eta_w$  and narrowing of the empennage  $\eta_e$ ;

– relative area of horizontal empennage  $S_{h.e}$  and relative area of vertical empennage  $S_{v.e}$  and their distance from the mass center of aircraft  $L_{h.e}$ ,  $L_{v.e}$ ;

– midsection of fuselage and engine nacelles  $S_{m.f}$ ,  $S_{m.e}$ ;

– engine bypass ratio  $\Theta$ ;

– compression degree in the compressor  $\pi_c$ ;

– turbine inlet temperature  $T$ .

Considered circuit features and parameters give only a generalized representation of the aircraft shape. For each unit of aircraft, there are a number of additional parameters (factors), rational choice of which largely define perfection of aircraft image (shape). So, for a wing such factors may be size and shape of the wing dogtooth extension and wingtips, wing sweep on the trailing edge, deformation laws of the middle surface and wing twist, type of mechanization, etc. For fuselage – is a form of cross-sections along its length, shapes of pilots cockpit and air intakes, etc.

No less significant impact on characteristics of the aircraft, primarily to interference and balancing resistances, producing a mutual location and ways of connecting wing, fuselage, empennage and air intakes, chassis lay-out.

Increasing the number of unknown parameters in the initial design stage, during consideration of their interaction in the process of finding the optimal shape of the aircraft, can lead to improving characteristics of the aircraft, but this is due to increasing complexity of computing algorithm and difficulties of implementing it on a computer.

The number of parameters that are best characterize the shape of the aircraft and able to be determined and optimized, is 15 – 20, depending on the structure of the aircraft.

From the customer's point of view, aircraft defined by a set of characteristics that are given or defined by design specification. Thus, characteristics of passenger aircraft are [2]:

- of commercial load mass  $m_c$  or number of passengers  $N_p$ ;
- estimated flight range  $L_c$ ;
- class of base airfield ( $L_r$  – length of the runway,  $\sigma_s$  – soil strength of base airfield);
- cruising speed  $V_c$ .

The criterion for evaluating effectiveness of civilian aircraft is specific expenses, which are based on cost of tone-kilometers for passengers and cargo. This specific cost is calculated by the formula [2]

$$a = \frac{100A}{k_{com}m_{com}V_r}, \quad (1)$$

where  $A$  is the cost of operating the aircraft during flight hour, RUB/h;  $m_{com}$  is commercial load, corresponding to a given flight distance, t;  $k_{com}$  is commercial load ratio, taking into account average annual load incomplete due to seasonality of transportation;  $V_r$  is regular speed.

Analysis of this formula shows that civilian aircraft as so effective (cost of transportation lesser) as:

- greater the commercial load  $m_{com}$ ;
- greater regular flight speed  $V_r$ ;
- lesser operating cost of aircraft within one flight hour  $A$ .

These three outputs are cause of three possible ways of designing passenger and cargo aircraft:

- increasing of commercial load;
- increasing of regular flight speed;
- reducing of aircraft operating costs.

Flight range of aircraft may be determined by the expression [2]

$$L = 1020 \frac{KM}{C_c} \ln \frac{m_0}{m_1}, \quad (2)$$

where  $K$ ,  $M$  and  $C_c$  – average values of aerodynamic aircraft features during the flight, flight number  $M$  and specific fuel consumption of engines, correspondingly;  $m_0$  and  $m_1$  – initial (takeoff) and final (landing) values of the aircraft mass.

A numerical optimization method for main aircraft parameters on the example of choosing cruising speed value can be clearly solved in MathCAD system. The essence of the method is as follows [3]:

1. Input values of  $K$ ,  $KM$  and  $KM/C_c$  depending on the flight number  $M$  for aircraft, optimally designed for each of corresponding  $M$  number in MathCAD carried by formulas (3) and (4) – (7).

2. In MathCAD system we obtained corresponding graphs as shown in Fig. 1 for the corresponding formulas.

$$M = \begin{pmatrix} M_1 \\ M_2 \\ \dots \\ M_m \end{pmatrix}, \quad K = \begin{pmatrix} K_1 \\ K_2 \\ \dots \\ K_i \end{pmatrix}; \quad (3)$$

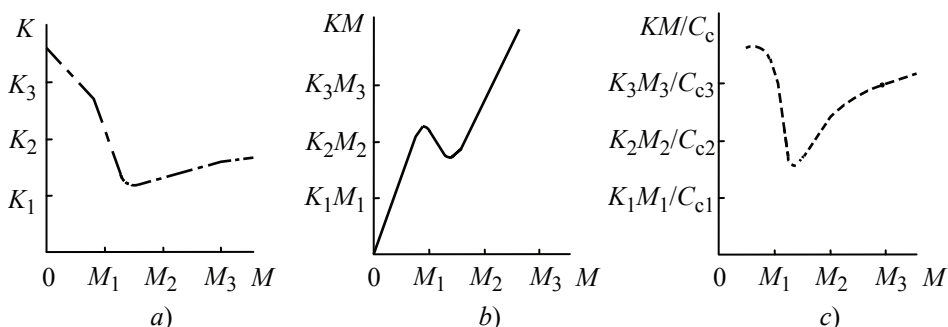
$$M = \begin{pmatrix} M_1 \\ M_2 \\ \dots \\ M_m \end{pmatrix}, \quad KM = \begin{pmatrix} K_1 M_1 \\ K_2 M_2 \\ \dots \\ K_i M_m \end{pmatrix}; \quad (4)$$

$$M = \begin{pmatrix} M_1 \\ M_2 \\ \dots \\ M_m \end{pmatrix}, \quad KM/C_c = \begin{pmatrix} K_1 M_1 / C_{c1} \\ K_2 M_2 / C_{c2} \\ \dots \\ K_i M_m / C_{cn} \end{pmatrix}; \quad (5)$$

$$KM = \text{cspline}(M, C_{cn}); \quad (6)$$

$$CP_n(M) = \text{interp}(M, C_{cn}, K_i), \quad (7)$$

where  $CP_n(M) = KM/C_c$ .



**Fig.1. Changing the values of  $K$  (a),  $KM$  (b) and  $KM/C_c$  (c) depending of number  $M$  while flight**

From Fig. 1 we can see:

- in the area of supersonic flight speeds drastically reduced the value of the aerodynamic characteristics of the aircraft;
- values  $(KM/C_c) \ln m_0/m_1$  are the same for flight numbers  $M$ , equals 1,1...1,2 and 2,0...2,5.

Let's take into account that operating efficiency of supersonic passenger aircraft is determined by following conditions:

- flight characteristics of aircraft should provide a high flight safety;
- aircraft must be adapted to existing equipment of airports and radio navigation services;
- operating costs and ticket prices should not more than 10 % higher than corresponding characteristics of transonic aircraft.

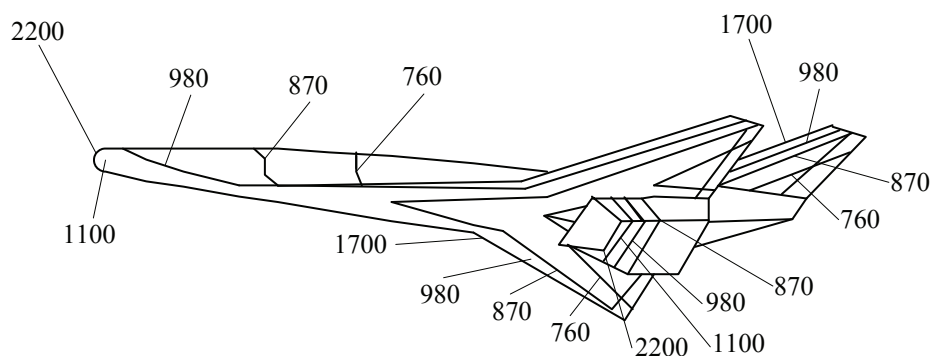
Usually you should optimize project of passenger supersonic aircraft by three parameters:

- cruising speed;
- flight range;
- number of passengers.

These parameters determine type of propulsion system, desired amount of fuel, aircraft structure, cost of its components, necessary equipment etc, i.e. determine take-off mass and cost of developing and operating the aircraft. Speed of the aircraft, which will replace operated transonic jets can't only slightly exceed their speed (800...1000 km/h), because it would be in a less advantageous range of Mach numbers, characterized by the appearance of a particularly large wave resistance. After passing this range, two favorable factors start to affect:

- aerodynamics of the aircraft improves;
- efficiency of turbojet engines increases.

Thus, cost-efficient passenger aircraft must fly at a rate much higher than sound speed. The level of development of modern science and technology allow to create a cost-effective passenger aircraft, with speeds of up to 3000 km/h. However, this needs to solve some important design and technological problems in the field of flight speeds at which disproportionately rapidly growing requirements for aircraft design and materials used. This applies primarily to the increase in temperature with increasing speed (Fig. 2) [4].



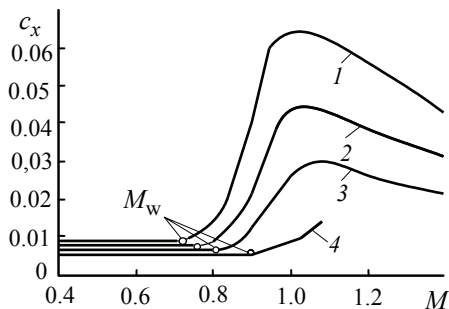
**Fig. 2. Steady-state temperature at the aircraft surface during a long flight, in Celsius degrees ( $V = 2400$  m/s;  $H = 34$  km)**

Aircraft passengers are not interested in the flight speed (a passenger does not feel speed if it is 1000 km/h, or 3000 km/h, and breaking the sound barrier does not render a significant physiological effect), but how much time he will spend on travel, using supersonic aircraft in the flight. Of course, increasing of the speed leads to a reduction in flight time (for passenger this means a reduction of travel time and for air transport enterprise – improving of transport efficiency), but it also depends on the non-stop flight distance.

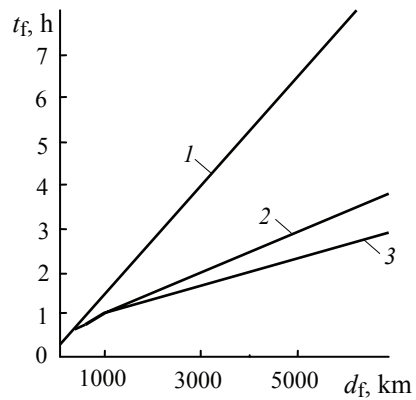
The dependences are shown in Fig. 3 for three types of aircraft based on assumption that time going to runway and waiting of takeoff is 15 min; acceleration and rising up to cruising altitude, as well as inhibition of supersonic aircraft before landing in the amount occupy 1000 km range. From Fig. 5 shows that for range of 2000 km saving of flight time with a cruising speed of 2125 km/h, compared with a transonic is 1 hr and 15 min, for range of 4000 km is 2 hrs 45 min, and for the range of 6000 km is more than 4 h. Reducing time of flight with the speed of 3200 km/h, relative to the previous, for same ranges are respectively only 10, 25 and 45 min, respectively [4].

Theoretical studies have shown that drag coefficient of supersonic passenger aircraft must be 3 times lesser than the typical value of this quantity for transonic aircraft. This is connected with choice of the appropriate aerodynamic configuration of the aircraft, and with definition of optimal forms of aircraft elements and characteristics of aircraft profiles for the given cruising speed. For passenger aircraft have no requirements of high maneuverability; they should have the best performance in flight at a constant speed and, in their design, focuses on maximizing aerodynamic efficiency at cruising speed. Range of flight for a given fuel volume or amount of required fuel and takeoff mass for fixed flight range depends from the aircraft aerodynamic quality.

The first method is to select a profile with small relative thickness (Fig. 4) [5].



**Fig. 4. Dependence  $c_x$  of direct wing from  $M$  number for different values of relative profile thickness:**  
1 – 0.10; 2 – 0.08; 3 – 0.06; 4 – 0.04

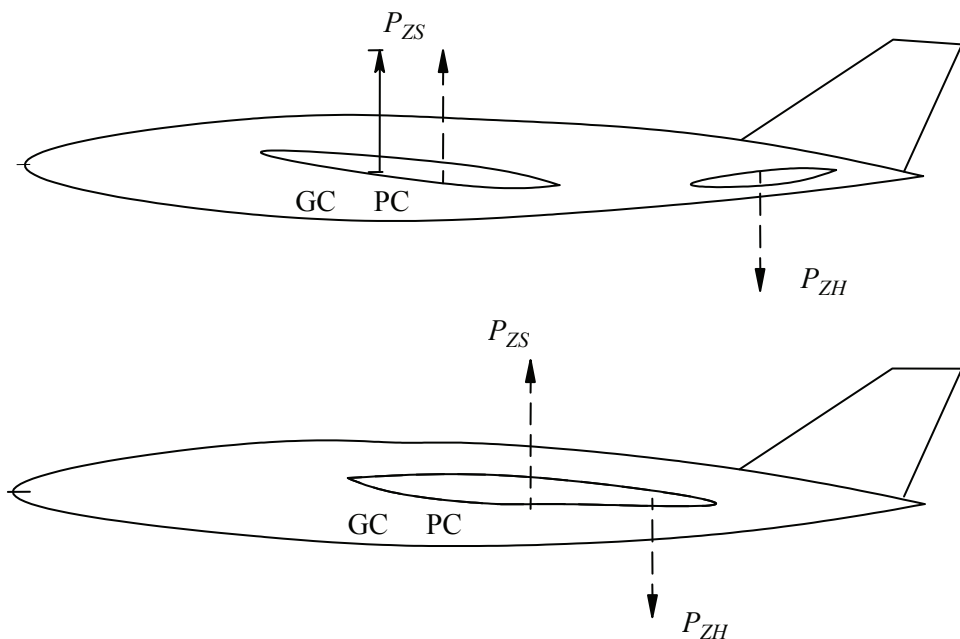


**Fig. 3. Dependence between duration  $t_f$  and distance of flight  $d_f$  for aircraft with different cruising speed, km/h:**  
1 – 870; 2 – 2125; 3 – 3186

Slim profiles have reduced lifting properties, but they simultaneously have very low resistance. Their using increases the aerodynamic efficiency of aircraft and reduces requirements for propulsion system. As shown in the figure, decrease in relative thickness of profile of the wing 4 up to 2.5 % gives quality increase by about 5 %. To realize benefits of thin profiles without increasing mass of the aircraft structure is necessary to use the delta stub wing. Small wingspan significantly reduces of bending moment, and a lot of overall

height in the root section allows creating a significant distance between power components, which leads to transformation of the bending moment in a couple of small quantities axial forces. These properties make it a rare example, which satisfy opposite requirements of high-speed aerodynamics and structural strength. The second way – is to reduce the surface, streamlined air flow, provided mainly by the choice of the fuselage with the minimum necessary volume and cross-section. Full surface of aircraft depends from aerodynamic design and, in particular, from presence or absence of horizontal stabilizer. It also affects on the value of balancing resistance.

When the distance between pressure center and gravity center increases, a longitudinal moment occurs, transforming flight mode into a dive (Fig. 5). To prevent this, it is necessary to balance the longitudinal moment by force  $P_{ZH}$ , created on control surfaces of horizontal empennage. The required amount of force  $P_{ZH}$  depends from arm on which it is applied, i.e. from selected aerodynamic configuration of the aircraft. In the classical scheme of the aircraft at subsonic speeds attitude  $P_{ZH}/P_{ZS}$  typically 0.03...0.05 and at supersonic increased to 0.15...0.20. This means that to balance aircraft flying at supersonic speeds it is necessary to increase the aerodynamic force of plumage in 4-5 times. Such balancing of aircraft associated with a significant increasing in resistance, because growth of force is provided by increasing of empennage deflection angle. This part of aircraft aerodynamic drag, called the trim drag, directly affect on aerodynamic quality changing. An aircraft without a horizontal stabilizer, parrying of longitudinal moment produce by elevons deviation. The center of pressure (PC) in such aircraft moved substantially smaller, but because of the small distance from the Gravity Center (GC), elevons should deviate at a larger angle.



**Fig. 5. Aerodynamic parrying of longitudinal moment**

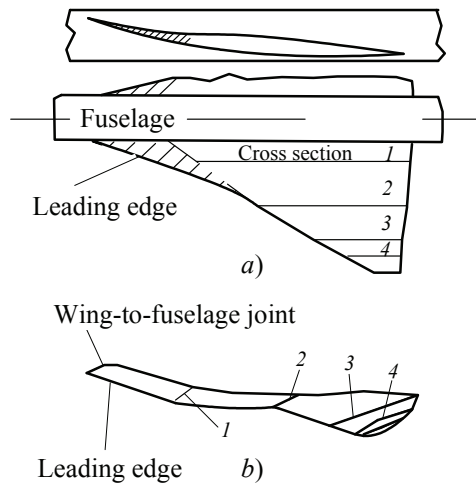
Growth of the trim drag, is also increase fuel consumption, and the problem is solved as limitation on moving of pressure center, and movement of the gravity center in the same direction, when it needs.

Application of wing consisting of two wings (surfaces) (Fig. 6) allows working on basic triangular surface with rounded ends on low speeds. Additional front part (striation) is very small elongation and a large sweep in such conditions almost does not lift. Only at high supersonic speeds sharply increases its effectiveness, so that resulting lifting force on it compensates the rearward displacement of pressure

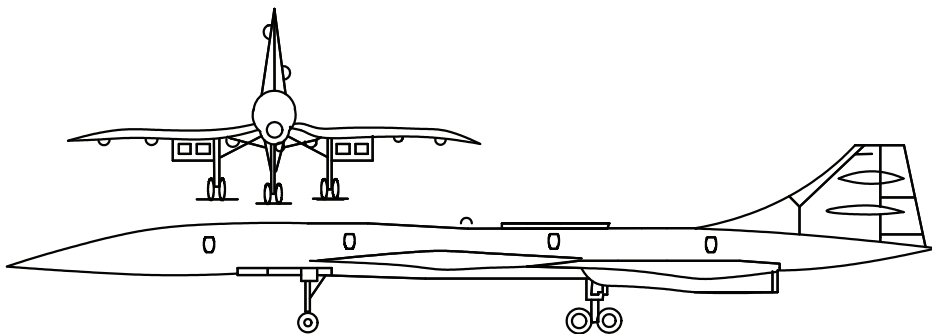
center of the basic part of triangular wing. The interaction of these two parts of the wing during flight can significantly reduce the displacement of pressure center during the transition from subsonic to supersonic flight speed (Fig. 6). Wings with a curved median surface and variable leading edge swept significantly increase aerodynamic efficiency of aircraft compared with wings used previously. Deformation wing by manner given in the Fig. 6, ensuring the aircraft at cruising speed characteristics of supersonic aircraft, and during takeoff and landing – the characteristics of subsonic aircraft.

Additional drag reduction can be achieved through using of slim fuselage (Fig. 7). Figure 7 shows the fuselage of a passenger aircraft without pilots cockpit and without windows of passenger cabin. All these design features can reduce drag of aircraft in all flight modes. For aircraft control is used system of sensors, signals are processed with modern computer and appropriate information is delivered to cockpit on monitors.

Additional resistance to interference from the engine nacelles and wing can be used as a factor with positive effect on lift of the aircraft.

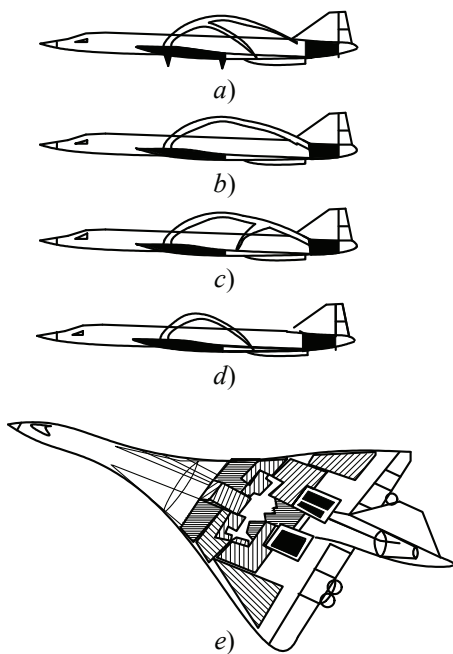


**Fig. 6. Example of wing consisting of two wings (surfaces):**  
*a* – wing root; *b* – general view ogival warped wing



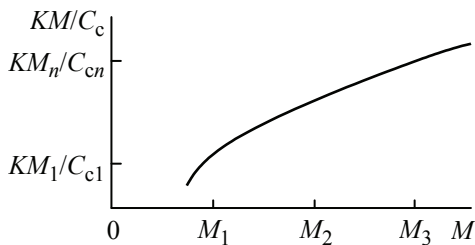
**Fig. 7. Example of slim fuselage**





**Fig. 8. Location of tanks and fuel pumping sequence depending on flight mode for «Concord» aircraft:**

- a – transition from subsonic to supersonic speed;
- b – braking; c – last stage of braking and transition to subsonic speed;
- d – pumping fuel from balancing tanks;
- e – general view



**Fig. 9. Changing in value  $KM/C_c$  from number  $M$  in flight, taking into account constructive solutions described above**

Solving the optimization problem by numerical method, considering all design solutions described above and substituting the resulting values into the formula (2) we obtain Fig. 9.

From Fig. 2 – 5 it is seen that applying of described above optimized solutions results in the value  $(KM/C_c)\ln(m_0/m_1)$  (according to formula (2)) increases.

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A characteristic feature of supersonic passenger aircraft is using of fuel (mass amount – 50 % of the takeoff mass of aircraft) for cooling the aircraft and for moving the center of gravity in transition from subsonic to supersonic flight speeds. This feature can be illustrated by example of the aircraft «Concord» (Fig. 8), which wings and fuselage have 17 fuel tanks, with total volume 117285 l. They are divided into three groups: the balancing tanks (4 in fuselage wing portion having a maximum sweep, and 1 at rear of the fuselage), reserve tanks (4 in wing) and main tanks (6 in wing and 2 in lower middle part of fuselage) [4].

Separation of internal space for each wing into seven separate fuel tanks – caissons required to ensure minimum possible displacement of aircraft gravity center as a result of fuel consumption and to control its position depending on the flight conditions. While taking off, lifting and transonic flight, front balancing tanks filled entirely, and rear tank is empty. In the transition from subsonic to supersonic flight speeds, fuel from front tanks is pumped into the back tank. As a result, gravity center moves rearward, i.e., moves followed the pressure center. In transition from supersonic to subsonic flight speeds, fuel pumped in opposite direction. Depending on the time of flight (consumed fuel) fuel from balancing tanks can be pumped into the main tanks.

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### Решение задачи оптимизации основных параметров самолета численным методом

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**Ключевые слова и фразы:** взлетная масса; крейсерская скорость; оптимизация численным методом; основные параметры самолета; параметрический анализ; себестоимость решения; целевая функция.

**Аннотация:** Проведен анализ оптимизационных задач на примере формирования облика транспортного самолета на этапе разработки технических предложений. Использован численный метод для решения задачи оптимизации основных параметров самолета. Численный метод решения основан на применении системы MathCAD. Используемый критерий при решении поставленной задачи – критерий «стоимость – эффективность». Рассмотрены два вектора оптимизации основных параметров самолета, с одной стороны – аэродинамические и энергетические параметры, с другой стороны – параметры, устанавливаемые заказчиком (компания, организация, эксплуатирующая проектируемый самолет).

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